RECREATIONS
FOR
GENTLEMEN *and* LADIES:
BEING INGENIOUS
SPORTS
AND
PASTIMES:

CONTAINING
Many curious Inventions, pleaſant Tricks on *Cards* and *Dice*; Arithmetical Sports; new Games; Rules for aſſuredly winning at all Games, whether of Cards or Dice; Recreative Fire-works; Tricks to promote Diverſion in Company, and other Curioſities.

Tranſlated from the FRENCH, of
MONSIEUR OZANAM.

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SPORTS AND PASTIMES.

Arithmetical Sports.

SPORT I.

Three jealous Huſbands with their wives being ready to paſs by night over a river, they find at the river ſide a boat which can carry but two perſons at once, and for want of a waterman they are neceſſitated to row themſelves over the river at ſeveral times: The queſtion is, how thoſe ſix perſons ſhall paſs 2 by 2, ſo that none of the 3 wives may be found in the company of 1 or 2 men, unleſs her huſband be preſent.

THEY muſt paſs in this manner, *viz.* firſt two women paſs, then one of them brings back the boat, and repaſſes with the third woman; that done, one of the three women brings back the boat, and ſitting down upon the ground with her huſband, permits the other two men to paſs over to find their wives: then one of the ſaid men with his wife brings back the boat, and placing her upon the ground, he takes the other man, and repaſſes with him: Laſtly, the woman who is found with the three men enters the boat, and at twice goes to fetch over the other two women.

SPORT II.

A Countryman having a fox, a gooſe, and a peck of corn, in his journey came to a river, where it ſo happened that he could carry but one over at a time. Now, as no two were to be together that might deſtroy each other; ſo he was at his wit's end how to diſpoſe of them; for, ſays he, though the corn can't eat the gooſe, nor the gooſe eat the fox, yet the fox can eat the gooſe, and the gooſe eat the corn: The queſtion is, how he muſt carry them over.

FIRST he muſt carry over the gooſe, leaving the fox and corn, (for the fox will not eat the corn) then, returning back, he may carry over the fox, bringing the gooſe back again; then leaving the gooſe, he may carry over the corn; laſtly, he muſt return to fetch the gooſe.

SPORT

SPORT III.

Two merry companions are to have equal ſhares of eight gallons of wine, which are in a veſſel containing exactly eight gallons: now to make this equal partition, they have only two other empty veſſels, of which one contains five gallons, and the other three; the queſtion is, how they ſhall exactly divide the wine by the help of thoſe three veſſels?

TO anſwer this queſtion; let us call the veſſel of 8 gallons A, the 5 gallon veſſel B, and the three gallon veſſel C. We ſuppoſe there are 8 gallons of wine in the veſſel A, and the other two, B and C, are empty, as you ſee in D. Having filled the veſſel B with wine out of the veſſel A, in which there will then remain but 3 gallons, as you ſee at E; fill the veſſel C with wine out of the veſſel B, in which, by conſequence, there will then remain but 2 gallons, as you ſee at F. This done, pour the wine of the veſſel C into the veſſel A, where there will then be 6 gallons, as you ſee in G; and pour the 2 gallons of the veſſel B into the veſſel C, which will then have 2 gallons, as you ſee at H; then fill the veſſel B with wine out of the veſſel A, by which means there will remain but 1 gallon in it, as you ſee

	8	5	3
	A	B	C
D	8	0	0
E	3	5	0
F	3	2	3
G	6	2	0
H	6	0	2
I	1	5	2
K	1	4	3

at I; and conclude the operation by filling the veſſel C with wine out of the veſſel B, in which there will then remain juſt 4 gallons, as you ſee at K; and ſo the queſtion is ſolved.

If, inſtead of the veſſel B, you would have the 4 gallons to remain in A, which we ſuppoſed to be filled with 8 gallons; fill the veſſel C with wine, out of the veſſel A, and ſo there will remain but 5 gallons in it, as you ſee at D; pour the 3 gallons of the veſſel C into the veſſel B. which will then have 3 gallons of wine, as you ſee at E; and having again filled the veſſel C with wine out of the veſſel A, where there will then remain but 2 gallons, as you ſee at F; fill up the veſſel B with wine out of C, where there will then remain but 1 gallon, as you ſee at G; at laſt, having poured the wine of the veſſel B into the veſſel A, where there will then be 7 gallons, as you ſee at H; pour the gallon of wine that is in C into the veſſel B, which by conſequence will have only 1 gallon, as you ſee at I; fill the veſſel C with wine out of the veſſel A, where there will then remain juſt 4 gallons, purſuant to the demand of the queſtion, as you ſee at K.

	8	5	3
	A	B	C
	8	0	0
D	5	0	3
E	5	3	0
F	2	3	3
G	2	5	1
H	7	0	1
I	7	0	1
K	4	1	3

SPORT IV.

A blind Abbeſs, viſiting her Nuns, who were equally diſtributed in eight cells built at the four corners of a ſquare, and in the middle of each ſide; finds an equal number of perſons in each row or ſide containing three cells: At a ſecond viſit, ſhe finds the ſame number of perſons in each row, though their number was enlarged by the acceſſion of four men: And coming a third time, ſhe ſtill finds the ſame number of perſons in each row, though the four men were then gone, and had carried each of them a nun with them.

TO reſolve the firſt caſe, when the 4 men were got into the cells, we muſt conceive it ſo, that there was a man in each corner cell, and that two nuns removed from thence to each of the middle cells: At this rate, each corner cell contained one perſon leſs than before, and each middle cell 2 more than before; ſuppoſe then that at the firſt viſitation, each cell contained three nuns, and ſo that there were 9 in each row, and 24 in all; at the ſecond viſit, which is the firſt caſe in queſtion, there muſt have been 5 nuns in each middle cell, and 2 perſons, *viz.* 1 man and 1 nun in each corner cell; which ſtill makes 9 perſons in each row.

3	3	3
3		3
3	3	3

2	5	2
5		5
2	5	2

4	1	4
1		1
4	1	4

To account for the ſecond caſe, when the 4 men were gone, and 4 nuns with them; each corner cell muſt have contained 1 nun more than at the firſt viſit, and each middle cell 2 fewer: And thus according to the ſuppoſition laid down, each corner cell contained 4 nuns, and there was only 1 in each middle cell; which ſtill make 9 in a row, though the whole number was but 20.

Note, Though at firſt ſight it may be thought by ſome that the four foregoing Sports cannot be reſolved by any certain rule, but only by many trials; yet by infallible argumentation and diſcourſe, the ſolution of thoſe queſtions may be found out, or elſe the impoſſibility of them, if by chance they ſhould be propounded impoſſible.

SPORT V.

Fifteen Chriſtians and fifteen Turks being at ſea, in one and the ſame ſhip in a terrible ſtorm, and the pilot declaring the neceſſity of caſting the one half of thoſe perſons into the ſea, that the reſt might be ſaved; they all agreed, that the perſons to be caſt away ſhould be ſet out by lot after this manner, viz. the thirty perſons ſhould be placed in a round form like a ring,

ring, and then beginning to count at one of the paſſengers, and proceeding circularly, every ninth perſon ſhould be caſt into the ſea, until of the thirty perſons there remained only fifteen. The queſtion is, how thoſe thirty perſons ought to be placed, that the lot might infallibly fall upon the fifteen Turks, and not upon any of the fifteen Chriſtians?

FOR the more eaſy remembring of the rule to reſolve this queſtion, I ſhall pre-ſuppoſe the five vowels, *a*, *e*, *i*, *o*, *u*, to ſignify five numbers, to wit, *(a)* one, *(e)* two, *(i)* three, *(o)* four, *(u)* five; then will the rule itſelf be briefly comprehended in theſe two following verſes:

From Numbers, Aid, and Art,
Never will fame depart.

In which verſes you are principally to obſerve the vowels, with their correſpondent numbers before aſſigned; and then beginning with the Chriſtians, the vowel *o* [in *from*] ſignifies that four Chriſtians are to be placed together; next unto them the vowel *u* [in *Num.*] imports that five Turks are to be placed together; in like manner *e* [in *bers*] denotes two Chriſtians; *a* [in *Aid*] one Turk; *i* [in *Aid*] three Chriſtians; *a* [in *and*] one Turk; *a* [in *Art*] one Chriſtian; *e* [in *ne*] two Turks; *e* [in *ver*] two Chriſtians; *i* [in *will*] three Turks; *a* [in *Fame*]

Fame] one Chriſtian; *e* [in *Fame*] two Turks; *e* [in *de*] two Chriſtians; *a* [in *part*] one Turk.

The invention of the ſaid rule, and ſuch like, depends upon the ſubſequent proceſs, *viz.* If the number of perſons be thirty, let thirty figures or cyphers be placed circularly, or elſe in a right line as you ſee,

oooooooooooooooooooooooooooooo

That done, begin to count from the firſt, and mark the ninth, (or what other ſhall be aſſigned) by putting a point or croſs over it; then count forward from that which you have marked, and place another point over the next ninth; and continue to do the ſame, beginning again when you ſhall be at the end (if the cyphers are placed in a right line) and paſſing over thoſe which you had already marked, until you have marked the number required, as in the example propounded, until you have marked fifteen; for then all the cyphers marked ſhall be thoſe which muſt be caſt away, and the others thoſe that are to remain. Hence it is evident, that if you obſerve how thoſe cyphers marked, are diſpoſed among thoſe which are not marked, you will eaſily make a rule for any number whatſoever.

By this invention (as ſome conjecture) the famous hiſtorian, *Joſephus* the *Jew*, preſerved his life very ſubtilly in the cave, to which himſelf and forty of his countrymen had fled from the furious and conquering *Romans* at the ſiege of *Jotapata:* For his ſaid countrymen having moſt wickedly reſolved to kill one another, rather than yield to their enemies, he at length (when no arguments that he could uſe, would diſſuade them from ſo horrid an act) prevailed with them to execute their tragical deſign by lot; and ſo by the help of the aforeſaid artifice, as we may ſuppoſe, himſelf, with one other perſon only remained alive, after the reſt were inhumanly murdered, they agreed to put an end to the lot, and thereby ſave their lives. This ſtory you may ſee at large in the fourteenth chapter of the third book of the hiſtory of *Joſephus*, of the wars of the Jews.

SPORT VI.

To ſubſtract, with one ſingle operation, ſeveral ſums, from ſeveral other ſums given.

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 56243
 84564
  3252 A
 26848
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  2942
  3654 B
  2308
--------
162003
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TO ſubſtract all the ſums that are under the line at B, from all the ſums above the line at A; begin by adding the numbers of figures of the right-hand column under the line, ſaying, 8 and 4 is 12, and 2 makes 14; which taken from the neareſt tens, *viz.* 20, there remains 6; which we add to the correſponding column above, ſaying 6 and 8 make 14, and 2 is 16, and 4 make 20, and 3 make 23: here we write 3 underneath; and in regard there are juſt two tens, as before, we retain or carry nothing. This done, we add after the ſame manner, the numbers of the next lower column, ſaying, 0 and 5 is 5, and 4 make 9; which taken from the neareſt ten, leaves 1; which we add, as above, to the ſuperior correſpondent column, ſaying, 1 and 4 make 5, and 5 make 10, and 6 make 16, and 4 make 20: here we ſet 0 underneath, and there being here two tens, whereas in the inferior correſponding column there was but one, we keep or carry the difference 1 to be taken from the next inferior column, becauſe we found more tens in A than in B: For had we found fewer in A than in B, we muſt have added the difference: and if it ſhould ſo fall out, that this difference can not be taken from the inferior column, for want of ſignificant figures, as it happens here in the fifth column; we muſt add it to the ſuperior column, and write the whole ſum under the line. Thus in the example propoſed, we have 162003, for the remainder of the ſubſtraction.

SPORT VII.

Compendious ways of MULTIPLICATION.

TO multiply any number, 128 for inſtance by a number that's the product of the multiplication of two other numbers; 24 for inſtance, the product of the multiplication of 4 and 6, or of 3 and 8: we multiply the number propoſed 128 by 4, and the product 512 by 6, (or elſe 128 by 3, and the product by 8) and have 3072 for the required multiplication.

Hence it follows, that to multiply a number propoſed by a ſquare number, we muſt multiply the number propoſed by the ſide or the root of the ſquare, and then the product by the ſame ſide again. Thus to multiply 128 by 25, we multiply it by 5, and the product by 5 again.

To multiply any number, 128 for inſtance, by a number that's the product of the multiplication of *three* other numbers, as 108 the product of 2, 6, and 9, or of 3, 6, and 6: we multiply 128 by 2, the product by 6, and the ſecond product by 9; or elſe 128 by 3, the product by 6, and the ſecond product by 6.

The conſequence of this is, that to multiply any number propoſed, by a cube-number, we muſt multiply it firſt by the ſide or root of the cube; then the product of that multiplication by the ſame root, and the ſecond product by the ſide again. As, to multiply 128 by 125, the cube-root of which is 5, we multiply 128 by 5, and the product 640 by 5 again, and the ſecond product 3200 by 5 again. Thus to find how many cubical feet are in 32 cubical toiſes, we multiply 32 by 6, the product of that by 6, and the ſecond product by 6.

To multiply any number by what power you will of five, add to the number propoſed, on the right-hand, as many cyphers as the exponent of the power contains unites, as, one cypher for 5, two for its ſquare 25, three for its cube 125, and ſo on: and divide the numbers thus augmented by the like power from 2, that is, 2

5, 4 for its ſquare 25, 8 for its cube 125, and ſo on.

Thus to multiply 128 by 5, we divide 1280 by 2, and the quotient 640 is the product of the multiplication: but to multiply 128 by 25 the ſquare of 5, we divide 12800 by 4 the ſquare of 2, and the quotient is the product demanded; and to multiply the ſame number 128 by 125 the cube of 5, we divide 128000 by 8 the cube of 2. And ſo on.

To know how many inches are in 53 foot, we multiply 53 by 12; or it might be done by multiplying 53 by 2, and the product by 6; or 53 by 3, and the product by 4. But there's a way of doing it without any multiplication; *viz.* by ſetting down 53 under 53, and then 53 again under both, advancing it a column to the left, ſo as to make 3 ſtand under 5; for the ſum of theſe three is 636, the number of inches contained in 53 foot, or of pence in 53 ſhillings.

$$\begin{array}{r} 53\\ 53\\ 53\\ \hline 636 \end{array}$$

To multiply together two numbers compoſed of ſeveral figures, 12, for inſtance, and 18; we reduce the firſt number 12, into theſe three parts, each of which conſiſts only of one figure, 2, 4, and 6; and in like manner the ſecond number, 18, into 4, 6, 8; each of which laſt muſt be multiplied by 2, the firſt part of the firſt number; and then by 4, the ſecond figure of the ſame firſt number; and at laſt by 6, the third part: and the ſum of all theſe products anſwers the demand.

SPORT VIII.

Diviſion ſhortened.

TO divide a large number by a ſmaller, by only addition and ſubſtraction, as 1492992 by 432; we commonly put the diviſor to the left, under 1492, to know how many times 'tis contained in that number. But yet we may ſave ourſelves that labour by making a tariff of the diviſor; for which end we place it on the right over againſt 1; then add to itſelf, or double it, and place that over againſt 2: Then we add it to the double, and place the ſum oppoſite to 3; adding it to the triple, we have its quadruple oppoſite to 4; as the additional of itſelf to the quadruple, gives the quintuple oppoſite to 5; and ſo of the other multiplies oppoſite to 6, 7, 8, 9, 10: The laſt of which, *viz.* the multiple correſponding to 10, ought, if the table is right done, to be the ſingle diviſor with a cypher on the right-hand.

1	432	1492992	(3456
2	864	1296 . .	
3	1296	————	
4	1728	1969	
5	2160	1728	
6	2592	——	
7	3024	2419	
8	3456	2160	
9	3888.	——	
10	4320	2592	
		2592	
		——	
		000	

Having thus prepared your table, proceed in the common way of diviſion; and every time you have occaſion to know how often your diviſor is contained in the correſponding number, look in your table for the neareſt number that does not exceed; and the number to which that is oppoſite gives you at one view the figure you are to put in your quotient. As, in the beginning of the diviſion here exemplified, you want to know how often 432 is to be found in 1492; in your table, you find 1296 (the neareſt number to 1492, and not exceeding it) oppoſite to 3, and accordingly 3 is the firſt figure of your quotient; and ſo of all the reſt.

This way is very convenient, when we have occaſion to divide large numbers by a ſmaller number; for the tairiff of our diviſor keeps us from being at a ſtand, by reſolving us readily upon all our diviſions. This is frequently the caſe of ſurveyors of land, who have occaſion to divide large numbers by 144, when they want to reduce ſquare inches into ſquare feet: or by 1728, when they want to reduce cubical inches into cubical feet.

To divide any number by what power you will of 5, multiply it by the like power of 2, and cut off from the right hand of the product as many figures as there are unites in the degree of the power; the remaining figures on the left will repreſent the quotient of the diviſion, and thoſe ſtruck off will be the numerator of a fraction, the denominator of which will be the like power of 10.

To divide any number by a ſmaller, that is the product of the multiplication of two yet ſmaller numbers, divide the number propoſed, by one of the two ſmaller, and the quotient by the other; and the ſecond quotient ariſing from the laſt diviſion, is what you want.

Thus to divide 20736 by 24, the product of 3 and 8, or of 4 and 6, we take the 8th part of its 3d, or the 6th part of its 4th, or, (which is the ſame thing) we take the third of its 8th part, or the 4th of its 6th, and our quotient proves 1728.

Hence to reduce ſquare feet to ſquare toiſes, (a toiſe is ſix foot) we muſt take the 6th part of the 6th part of the number propoſed of ſquare feet, becauſe a ſquare toiſe is 36 ſquare feet, and 6 times 6 is 36. Thus to reduce 542 ſquare feet to ſquare toiſes, we muſt take the 6th part of $90\frac{2}{6}$ (the 6th part of 542) and ſo have 15 ſquare toiſes and 2 ſquare feet, as the value of 542 ſquare feet.

SPORT IX.

Curious properties of Numbers.

I. NUMBER 9 has this property; that when it multiplies any number of integers whatſoever, the ſum of the figures in the product is diviſible by 9: thus 53, multiplied by 9, makes the product 477; the figures of which, added together, *viz.* 7 and 7 and 4 make 18, which is exactly diviſible by 9.

II. Take any two numbers whatſoever, either one of the two, or their ſum, or their difference is diviſible by 3: thus, of the two numbers 6 and 5, 6 is diviſible by 3; of 11 and 5, the difference 6 is diviſible by 3; of 7 and 5 the ſum 12 is diviſible by 3.

III. The product ariſing from the multiplication of two numbers, the ſquares of which make a joint ſquare number, is diviſible by 6: thus 12 the product of 3 and 4, the ſquares of which, *viz.* 9 and 16, make together the ſquare number 25; this 12, I ſay, is diviſible by 6.

To find two numbers, the ſquares of which make together a ſquare number, multiply any two numbers, the one by the other, and the double of the product will be one of the two numbers demanded, and the difference of their ſquares will be the other. Thus in 2 and 3, the double of their product 12, and 5 the difference of their ſquares (4 and 9) are two numbers of that quality, that their ſquares 144 and 25 make together the ſquare number 169, the root of which is 13. *See* Sport X *and* XI.

IV. The ſum and the difference of any two numbers, the ſquares of which differ by a ſquare number, are each of them, either a ſquare number, or the half of one; thus, take the number 6 and 10, their ſquares 36 and 100 differ by the ſquare number 64; their ſum is 16, and their difference 4, each of which is a ſquare number: then take 8 and 10 for the two numbers, their ſquares 64 and 100, differ by the ſquare number 36; and the ſum 18, and the difference 2, are the halfs of the two ſquare numbers 36 and 4.—

To find two numbers, the ſum and the difference of which, are, each of them, a ſquare number, In which caſe, the ſquares of theſe two numbers will likewiſe differ by a ſquare number; pitch upon any two numbers, as 2 and 3, the product of their multiplication is 6, their ſquares are 4 and 9: 13 the ſum of the two ſqures, and 12 the double of the product of their multiplications, are the numbers we look for; for their ſum 25, and their difference 1 are both ſquare numbers; and further, their ſquares 169 and 144 differ by the ſquare number 25.

To find two numbers, the ſum and difference of which are each of them the half or the double of a ſquare number; in which caſe their ſquares will likewiſe differ by a ſquare number; take any two numbers, as 2 and 3, the ſquares of which are 4 and 9; 13 the ſum of theſe two ſquares, and 5 the difference, are the two numbers demanded; for their ſum 18 and their difference 8, are the halfs of the two ſquare numbers 36 and 16, and the doubles of the two ſquare numbers 9 and 4; and farther, their ſquares 169 and 25, differ by the ſquare number 144, the root of which is 12.

V. Every ſquare number ends either with two cyphers, or with one of the 5 figures 1, 4, 5, 6, 9, which ſerves for a rule, *to diſtinguiſh when a number propoſed is not ſquare*, viz. when it does not end as above; nay, if it does end with two noughts, and theſe are not preceded by any of the foregoing five figures, we may reſt aſſured it is not ſquare.

VI. Every ſquare fraction, that is, every fraction that has its ſquare root, is ſuch, that the product of the multiplication of the numerator by the denominator is ſquare. Thus *we know a fraction is not ſquare,* when that does not happen. Take the fraction $\frac{28}{63}$, we know it to be ſquare, becauſe 1764, the product of 28, multiplied by 63, is a ſquare number, having 42 for its root: and ſo the ſquare root of the propoſed fraction is $\frac{42}{63}$, retaining the ſame denominator; or $\frac{28}{42}$, retaining the ſame numerator, for either of theſe is equivalent to $\frac{2}{3}$, for the ſquare root of the propoſed fraction $\frac{28}{63}$ or $\frac{4}{9}$.

VII. Any cubical fraction, *i. e.* any that has its cube root, is ſuch, that if you multiply the numerator by the ſquare of the denominator, or the denominator by the ſquare of the numerator, the product has its cube-root; and it is by this rule that *we know when a fraction is a cube fraction, ſuch is* $\frac{24}{375}$ for 3375000, and 216000, the two products of the two ways of multiplication juſt mentioned, have 150 and 60 for their cube roots, and ſo the cube-root of the fraction $\frac{24}{375}$ is $\frac{150}{375}$ retaining the ſame denominator, or $\frac{24}{60}$ retaining the ſame numerator, for each of theſe fractions is equal to $\frac{2}{5}$ as the cube-root of the propoſed fraction $\frac{24}{375}$

VIII. Though it is not poſſible to find two homogeneous powers, the ſum and difference of which are each of them a power of the ſame degree, that is, ſquare numbers if the two firſt are ſquares, and cube numbers if they are cubical, &c. yet it is poſſible, and very eaſy, *to find two triangular numbers, the ſum and difference of which, are each of them a triangular number.*

Thus 15 and 21 are two triangular numbers, the ſides of which are 5 and 6, and their ſum 36; and the difference 6 are likewiſe triangular numbers, having 8 and 3 for their ſides. Again, 780 and 990 are triangular numbers, the ſides of which are 39 and 44, and their ſum 1770, and the difference 210 are likewiſe triangular numbers, having 59 and 20 for their ſides. Once more, 1747515 and 2185095 are triangular numbers, having 1869 and 2090 for their ſides; and their ſum 3932610 and the difference 437580 are likewiſe triangular numbers, the ſides of which are 2804 and 935.

By a *Triangular Number* we underſtand the ſum of the natural numbers, 1, 2, 3, 4, 5, 6, beginning with unit, and riſing to what number you will, the laſt and the greateſt of which is called the ſide. Thus we know that 10 is a triangular number, the ſide of which is four, by reaſon that it is equal to the ſum of the firſt four natural numbers, 1, 2, 3, 4, the laſt and greateſt of which is 4. It is called *Triangular*, becauſe you may diſpoſe 10 points in the form of an equilateral triangle, each ſide of which contains 4, and hence it was that 4 got the name of the ſide of the triangular number 10.

To know if a number propoſed is triangular, you muſt multiply it by 8, and add 1 to the product, for if the ſum be ſquare, the propoſed number is triangular. Thus we know that 10 is triangular, becauſe 81 (the ſum of its multiplication by 8, with the addition of 1) is a ſquare number, having 9 for its root.

IX. The difference of two homogeneous powers, as of two ſquare numbers, of two cube numbers, *&c.* is diviſible by the difference of their ſides. Accordingly we find that 21, the difference of the two ſquare numbers 25 and 4, the ſides of which are 5 and 2, is diviſible by 3, the difference of the ſides or roots, the quotient 7 being always equal to the ſum of the ſame ſides or roots; and that 117, the difference of the cubes 125 and 8, the roots of which are 5 and 2, is diviſible by 3 the difference of the roots, the quotient 39 being equal to the product of the ſaid roots multiplied one into another, *viz.* 10 added to 29, the ſum of their ſquares 25 and 4.

X. The difference of the two homogeneal powers, the common exponent of which is an even number, is diviſible by the ſum of their roots. Thus, 21 the difference of the two ſquare numbers 25 and 4, the roots of which are 5 and 2, is diviſible by 7, the ſum of the ſaid roots, the quotient 3 being equal to the difference of the roots, and 609 the difference of the bi-quadrats 625 and 16, the roots of which equal

which are 5 and 2, is diviſible by 7, the ſum of the roots, the quotient 87 being equal to the product ariſing from 3, the difference of the roots, multiplied with 29, the ſum of their ſquares 25 and 4.

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XI. The ſum of two homogeneal powers, the common exponent of which is an odd number, is diviſible by the ſum of their roots. Thus we know that 133 the ſum of the two cubes 125 and 8, the roots of which are 5 and 2, is diviſible by 7 the ſum of theſe roots, the quotient 19 being equal to the exceſs of the ſum of the ſquares of the roots (29) above the product of the roots (10). And that 3157 the ſum of the two ſur-ſolids 3125 and 32, the roots of which are 5 and 2, is diviſible by 7 the ſum of the roots; the quotient 451 being equal to the exceſs of 741 the ſum of the bi-quadrat powers of the roots 5 and 2 (625, 16) and of the ſquare of the product of the roots (100) its exceſs, I ſay, above 290 the product of the ſum of the ſquares of the ſame roots (29) multiplied by 10 the product of the roots themſelves.

XII. All the powers of the natural numbers 1, 2, 3, 4, 5, 6, &c. have as many differences as their exponents contain units, the laſt differences being always equal among themſelves in each power, that is, the ſecond differences, or the differences of the differences, in the ſquares 1, 4, 9, 16, 25, 36, &c. for theſe ſecond differences make 2, the firſt being the uneven numbers 3, 5, 7, 9, 11, &c. The third differences, or the differences of the differences of the firſt differences in the cubes 1, 8, 27, 64, 125, 216, &c. for theſe third differences make 6, the firſt being 7, 19, 37, 61, 91, &c. and the ſecond differences, *i. e.* the differences of theſe differences being 12, 18, 24, 30, &c. which riſe by 6 for the third difference, and ſo of the reſt.

The ſame thing happens to Pollygon numbers formed by the continual addition of numbers in continual arithmetical progreſſion, which are called Gnomons, and of which the firſt is always an unite, which is virtually any Polygon number. The ſame is the caſe with Pyramidal numbers, which are formed by the continual addition of Polygon numbers conſidered as Gnomons, the firſt of which is always unit: and in like manner with the Pyramido Pyramidal numbers, which are produced by the continual addition of Pyramidal numbers, conſidered as Gnomons, the firſt of which is always unity.

When the Gnomons riſe, or exceed one another by one, as 1, 2, 3, 4, 5, 6, &c. the Polygon numbers 1, 3, 6, 10, 15, 21, &c. which are formed from them are called Triangular, the property of which is ſuch, that each of them being multiplied by 8, and the product inlarged by unity, the ſum is a ſquare number, as we intimated above. And farther, 9 the ſum of the ſecond and the third, omitting the firſt, is a ſquare number, and 36 the ſum of the fifth and the ſixth, omitting the fourth, is likewiſe ſquare, and ſo on.

When the Gnomons riſe, or exceed one another by two unites, as the odd numbers 1, 3, 5, 7, 9, 11, &c. the Polygon numbers formed from them, 1, 4, 9, 16, 25, 36, &c. are ſquare numbers; and when the Gnomons increaſe by three unites, as 1, 4, 7, 10, 13, 16, &c. the numbers formed from them, 1, 5, 12, 22, 35, 51, &c. are called Pentagons, and have this peculiar quality, that each of them being multiplied by 24, and 1 added to the product, the ſum is a ſquare number, by which rule *we know when a propoſed number is Pentagon*, and ſo of the others.

To find the ſum of as many triangular numbers as you will, commencing from unit, of theſe eight for inſtance, 1, 3, 6, 10, 15, 21, 28, 36, multiply the given number 8 by the next follower 9, and the product 72, by the next after that 10, and divide the ſecond product 720 by 6, the quotient gives you 120 the ſum demanded.

The ſum of all theſe infinite fractions $\frac{1}{3}, \frac{1}{6}, \frac{1}{10}, \frac{1}{15}, \frac{1}{21}$, &c. the common numerator of which is 1, and the denominators of which are triangular numbers, their ſum I ſay, is juſt 1.

To find the ſum of as many ſquare numbers from an unit as you will, of theſe eight, for example, 1, 4, 9, 16, 25, 36, 49, 64, take 36, the laſt of as many triangular numbers, *viz.* 1, 3, 6, 10, 15, 21, 28, 36, from 240 the double of this ſum 120, and the remainder 204 is the ſum you want.

XIII. The cubes, 1, 8, 27, 64, 125, 216, *&c.* of the natural numbers, 1, 2, 3, 4, 5, 6, *&c.* are ſuch, that the firſt 1 is a ſquare number, the root of which 1 is the firſt triangular number; the ſum of the two firſt, 1 and 8, *viz.* 9 is a ſquare number, the root of which 3 is the ſecond triangular numbers 36, the ſum of the three firſt, 1, 8, 27, is a ſquare number, the root of which 6 is the third triangular number, and ſo on. And therefore if you want *to find the ſum of any number of cubic numbers from an unit,* of theſe ſix for example, 1, 8, 27, 64, 125, 216,) the ſquare of the ſixth triangular number 21, (441) is the ſum you deſire.

XIV. Among whole numbers, there is only 2 that being added to itſelf, makes as much as when multiplied by itſelf, *viz.* 4, for all other numbers make more by multiplication than by addition.

Though we cannot find two whole numbers the ſum of which is equal to the product of their multiplication, yet we can eaſily *find two fractional numbers, and even in a given Ratio, the ſum of which is equal to their product, viz.* by dividing the ſum of the two terms of the given Ratio by each of the two terms; thus, if you give them the Ratio of the two numbers, 2, 3, divide their ſum 5 ſeparately by 2 and by 3, and you will have the two numbers $2\frac{1}{2}$, $1\frac{2}{3}$, which make as much when added together as when multiplied together, *viz.* $4\frac{1}{6}$.

XV. Any number is the halt of the ſum of two others equally remote, the one in the way of defect, and the other in exceſs. For example, 6 is the half of 12, the ſum of the two numbers equally remote, 5 and 7, or 4 and 8.

XVI. The number 37 has this property, that being multiplied by any of theſe numbers, 3, 6, 9, 12, 15, 18, 21, 24, 27, which are in continual arithmetical progreſſion, all the products are compoſed of one figure thrice repeated.

37	37	37	37	37	37	37	37	37
3	6	9	12	15	18	21	24	27
111	222	333	444	555	666	777	888	999

XVII. The two numbers 5 and 6 are called Spherical, becauſe their powers terminate in theſe very numbers. The powers of 5, *viz.* 25, 125, 625, *&c.* terminate in 5, and in like manner the powers of 6, *viz.* 36, 216, 1296, *&c.* end with 6.

5 has that peculiar quality, that when multiplied by an odd number (as 7) its product terminates in 5, (as 35) and when multiplied by an even number (as 8) its product ends in a cypher, (as 40).

The other number, 6, has likewiſe this ſingular quality, that it is the firſt of the numbers which we call *perfect*, as being equal to the ſum of their aliquot parts, for 6 is equal to the ſum of its aliquot parts 1, 2, 3; 28 is likewiſe a *perfect* number, in regard it is equal to the ſum of its aliquot parts 1, 2, 4, 7, 14: and one may find an infinity of other perfect number, as 496, which is equal to the ſum of its aliquot parts 1, 2, 4, 8, 16, 31, 62, 124, 248.

To find all the perfect numbers in order, make uſe of the powers of 2, *viz.* 2, 4, 8, 16, 32, *&c.* and ſee which of theſe powers, when an unit is taken from them, makes a prime number, and you will find in 4, 8, 32, *&c.* that if you ſubſtract 1 from each of them, the remainders 3, 7, 31, *&c.* are prime numbers, each of which ought to be multiplied by the half of the correſponding power, that is, 3 by 2, 7 by 4, 31 by 16, *&c.* in order to obtain the perfect numbers 6, 28, 496, *&c.*

2	4	8	16	32
	1	1		1
	—	—		—
	3	7		31
	2	4		16
	—	—		—
	6	28		496

To find all the Aliquot Parts, or all the diviſors of a propoſed number, of which an unit is always one. If the number be 8128 (for example) which is likewiſe a perfect number, divide it by the leaſt number that offers, *viz.* 2, which is eaſily done, becauſe 8128 is an even number, ſo the quotient will be 4064, which ſet down over againſt 2 for your ſecond diviſor, which may ſtill be divided by the firſt diviſor 2, and ſo its ſquare 4 may likewiſe be a diviſor, which ſet down under 2, over againſt the ſecond quotient 2032 for another diviſor, which may ſtill be divided by the firſt diviſor 2, and therefore its cube 8, will likewiſe be a diviſor, which you are to write under the ſquare 4, and oppoſite to the third quotient 1016 for another diviſor: thus you go on, till you come to the laſt diviſor that cannot be divided by 2, *viz.* the ſixth quotient 127, which being a prime number, that is, a number that can be divided by nothing but an unit, gives us to know that we have traced all the diviſors of the number propoſed 8128, and here you ſee the ſum of the diviſors is equal to the number propoſed, and by conſequence it is a perfect number.

1	
2	4064
4	2032
8	1016
16	508
32	254
64	127
—	——
127	8001
	127
	——
	8128

By the ſame method did we find out all the diviſors of the other number 2096128 which is likewiſe perfect, for as you ſee it is equal to the ſum of its aliquot parts. You ſee likewiſe that the laſt quotient 2047 which anſwers to 1024 the tenth power of the firſt diviſor 2 is alſo a prime number, for if it could have been divided by any other number beyond 2, as by 3, it behoved us to have multiplied all the power of the firſt diviſor 2 by this new diviſor 3, and to have divided the number propoſed and all the quotient by this new diviſor 3, in order to have their diviſors, as you will ſee in the following example.

1	
2	1048064
4	524032
8	262016
16	131008
32	65504
64	32752
128	16376
256	8188
512	4094
1024	2047
——	—— ——
2047	2094081

XVIII.

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XVIII. The number 120 is equal to the half of 240, the ſum of its aliquot parts 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60. The number 672 is likewiſe equal to the half of 1344 the ſum of its aliquot parts, as will appear by obſerving the method above preſcribed, which we ſhall not now repeat. We may find a great many other numbers that have the ſame quality; nay, ſome may be found to be the third, or any other part of the ſum of their aliquot parts, which we ſhall not now inſiſt upon.

XIX. The two numbers 220 and 284 are called *Amiable*, becauſe the firſt 220 is equal to the ſum of the aliquot parts of the latter, 1, 2, 4, 71, 142; and reciprocally the latter 284 is equal to the ſum of the aliquot parts of the former, 1, 2, 4, 5, 10, 11, 22, 44, 55, 110. Theſe aliquot parts are eaſily found by what we have ſaid before, eſpecially if we conſider that all numbers that end in 5, or in 0, are diviſible by 5.

To find all the Amiable Numbers in order, make uſe of the number 2, which is of ſuch a quality, that if you take 1 from its tripple 6, from its ſextuple 12, from the octodecuple of its ſquare 72, the remainders are the three prime numbers 5, 11, and 71, of which 5 and 11 being multiplied together, and the product 55 being multiplied by 4 the double of the number 2, this ſecond product 220 will be the firſt of the two numbers we look for; and to find the other 284, we need only to multiply the third prime

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prime number 71 by 4, the ſame double of 2, that we uſed before.

To find two other amiable numbers, inſtead of 2 we make uſe of two of its powers that poſſeſſes the ſame quality, ſuch as its cube 8; for you ſubtract an unit from its tripple 24, from its ſextuple 48, and from 1152 the octodecuple of its ſquare 64, the remainders are the three prime numbers, *viz.* 23, 47, 1151, of which the two firſt, 23, 47, ought to be multiplied together, and their product 1081 ought to be multiplied by 16 the double of the cube 8, in order to have 17296 for the firſt of the two numbers demanded. And for the other amiable number demanded, which is 18416, we muſt multiply the third prime number 1151 by 16, the ſame double of the cube 8.

If you ſtill want other amiable numbers, inſtead of 2, or its cube 8, make uſe of its ſquare cube 64, for it has the ſame quality, and will anſwer as above.

In regard, it is difficult to know whether a number is prime if it be a large number, we ſhall at the end of this Sport ſubjoin a table of all the prime numbers between 1 and a 10000.

XX. The ſquares of the two numbers 31, 34, *viz.* 961, 1156, are ſuch, that the firſt 961, with its aliquot parts, 1, 31, makes a ſum (993) equal to 1, 2, 4, 17, 34, 68, 289, 578, the aliquot parts of the ſecond 1156.

XXI. The two numbers 26, 20, make each of them with their aliquot parts, the ſame ſum; the firſt 26 with its aliquot parts, 1, 2, 13, makes 42, and the ſecond (20) with its aliquot parts 1, 2, 4, 5, 10, makes likewiſe 42.

The ſame is the caſe of 488 and 464, each of them with their aliquot parts making 930: of 11 and 6, each of them with their aliquot parts making 12; and in fine of 17 and 10, which with their aliquot parts make 18 a piece.

Nay, we may find three numbers, each of which with its aliquot parts makes the ſame ſum, as 20, 26 and 41, as alſo 23, 14, 15 and 46, 51, 71.

We may find two ſquare numbers of the ſame quality, particularly 16 and 25 the ſquares of 4 and 5; which are the loweſt that can be, and by virtue of which we come at as many more as we will of the ſame quality, *viz.* by multiplying them by ſome odd ſquare number, that is not diviſible by five. For example, if we multiply each of them by the ſquare number 9, we obtain two other ſquare numbers 144 and 225, each of which with its aliquot parts makes juſt 403.

XXII. 81 the ſquare of 9, with its aliquot parts, 1, 3, 9, 27, makes a ſquare number (121) the root of which is 11, 400 the ſquare of 20, with its aliquot parts, make the ſquare of 31 (961).

XXIII. 666 the ſum of theſe three triangular numbers 15, 21, 630, the ſides of which are 5, 6, 35, is likewiſe a triangular number, the ſides of which is 36. The ſame is the caſe of theſe three triangular numbers 210, 780, 1711, and likewiſe of theſe 666, 2628, 5886.

XXIV. 49 the ſquare of 7 has this quality, that 8 the ſum of its aliquot parts, 1, 7, is the cube of 2, and 343 the cube of the ſame number 7, does with its aliquot parts 1, 7, 49, make the ſquare number 400, the root of which is 20. I do not here pretend to direct you how to find out others of the ſame quality, for unleſs you light upon them by chance, it is very difficult to trace them without Algebra, which I propoſe not to mention in this performance.

XXV. 9 the ſquare of 3 has this quality, that 4 the ſum of its aliquot parts 1, 3, is the ſquare of 2, 2401 the ſquare of 49 has the ſame quality, for 400, the ſum of its aliquot parts 1, 7, 49, 343 is the ſquare of 20.

XXVI. The two numbers 99, 63, have this quality, that (57) the ſum of the aliquot parts of the firſt, 1, 3, 9, 11, 33, ſurpaſſes (41) the ſum of the aliquot parts of the ſecond, 1, 3, 7, 9, 21, by the ſquare number 16, the root of which is 4. The ſame is the condition of 325 and 175; for the ſum of the aliquot parts of the firſt exceeds that of the aliquot parts of the other, by the ſquare number 36.

XXVII. The ſum of two numbers that differ by unity, is equal to the difference of their ſquares, and the ſum of the ſquares of their triangular numbers is likewiſe a triangular number. Thus 5 and 6 make the ſum 11 equal to the difference of their ſquares 25, 36, and their triangulor numbers 15, 21, are ſuch, that 666, the ſum of their ſquares 225, 441, is likewiſe a triangular number, the ſide of which is 36.

XXVIII. The two triangular numbers, 6, 10, of the two numbers 3, 4, the difference of which is likewiſe an unity, have this quality, that their ſum 16, and their difference 4, are ſquare numbers, having 4 and 2 for roots; and 136 the ſum of their ſquares (36, 100) is a triangular number, the ſide of which 16 is likewiſe a ſquare number, the root of which is at the ſame time a ſquare number, having 2 for its ſide or root.

The ſame is the quality of the two other triangular numbers, 36, 47 the ſides of which, 8, 9, differ only by unity, for their ſum 81, and their difference 9, are ſquare numbers, the roots of which are 9 and 3, and 3321 the ſum of their ſquares (1296, 2025) is a triangular number, the ſide of which is 81, and that has its ſquare root 9, which again is the ſquare of 3.

There are many other triangular numbers of this quality, that may be found out by ſubſtracting and adding any ſquare number to its ſquare, the halves of the remainder and of the ſum being the two triangular numbers demanded. For example, if you ſubſtract 8 the ſquare number 16 from and add it to its ſquare 256, half the remainder 240, and half the ſum 272, preſent us with 120, and 136, for the two triangular numbers thought for, the ſides of which are 15, 16, the difference conſiſting ſtill in unity.

Theſe two triangular numbers thus found, have this farther quality, that the greateſt of their ſides is always a ſquare number, and the difference of their ſquares is likewiſe a ſquare number; and withal their ſum is a bi quadrate, equal to the ſquare of their difference, and at the ſame time to the ſide of the triangular number that compoſes the ſum of their ſquares.

XXIX. The difference of the ſquares of the two numbers in a duplicate Ratio, is equal to the ſum of their cubes divided by the ſum of their two numbers, and that very ſum of their cubes is the third of a cube.

Accordingly, 4 and 8 being in a duplicate Ratio, the difference 48 of their ſquares, 16, 64, is equal to the quotient reſulting from the diviſion of 576 (the ſum of their cubes, 64, 512) by 12 the ſum of the two numbers, and the very ſum of their cubes 576 is the third part of the cube 1728, the root of which 12 is always equal to the ſum of the two numbers.

I ſhould never have done, if I pretended here to fetch in all the properties of numbers, which indeed are infinite, and upon that conſideration I ſhall now conclude this Sport with the table of the prime numbers that I promiſed above.

Table of the Prime Numbers between 1 *and* 10000.

2
3
5
7
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17
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43
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53
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61
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71
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83
89
97
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101
103
107
109
113
127
131
137
139

149
151
157
163
167
173
179
181
191
193
197
199
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211
223
227
229
233
239
241
251
257
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269
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293
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307
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383
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397
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401
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503
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541

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631
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653
659
661
673
677
683
691
—
701
709
719
727
733
739
743
751

757
761
769
773
787
797
—
811
821
823
827
829
839
853
857
859
863
877
881
883
887
—
907
911
919
929
937
941
947
953
961
971
977
983
991

997
—
1009
1013
1019
1021
1031
1033
1039
1049
1051
1061
1063
1069
1087
1091
1093
1097
—
1103
1109
1117
1123
1129
1151
1153
1163
1171
1181
1187
1193
—
1201
1213
1217

1223
1229
1231
1237
1249
1259
1277
1279
1283
1289
1291
1297
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1301
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1307
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1327
1361
1367
1373
1381
1399
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1409
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1451
1453
1459

1471
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1483
1487
1489
1493
1499
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1511
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1543
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1561
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1583
1597
—
1601
1607
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1613
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1637
1657
1663
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1669
1693
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1933
1949
1951

1973
1979
1987
1993
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2003
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SPORT XI.

Of Right-angled Triangles in numbers.

BY a Rectangular Triangle in numbers, we mean three unequal numbers, the greateſt of which is ſuch, that its ſquare is equal to the ſquare of the other two, ſuch are 3, 4, 5, for 25 the ſquare of 5 the greateſt, which we call the Hypothenuſe, is equal to the ſum of 9 and 16, the ſquares of the other two numbers, 3, 4, which we call the ſides, taking one for the Baſe of the right-angled triangle, and the other for the Altitude or Height. Half the product of the Baſe, and the Altitude, is called the Area, and is always diviſible by 3. The reader will obſerve all along that by the product of two numbers, we underſtand the number ariſing from their mutual multiplication.

There is an infinite number of right-angled triangles of divers ſorts, both in whole and in broken or fractional numbers, but we generally conceive them in integers, among which the firſt and the leaſt of all is that now mentioned, 3, 4, 5, which has an infinity of fine properties, but it would be tedious to enumerate them, and therefore I ſhall content myſelf with obſerving, that the ſum (216) of the cubes (27, 64, 125) of the two ſides 3, 5, and of the Hypothenuſe (5) is a cube, the root or ſide of which (6) is equal to its Area.

To find in numbers as many Right-angled Triangles as you will: Take any two numbers, for example 2 and 3, which we call Generating Numbers, multiply them the one by the other, and (12) the double of their product (6) is the ſide of a right-lined triangle, the other ſide being equal to (5) the difference of the ſquares (4, 9) of the generating numbers 2, 3, and the Hypothenuſe being equal to (13) the ſum of the ſame ſquares, 4, 9. And thus you have this right-angled triangle 5, 12, 13, for 169, the ſquare of the Hypothenuſe 13 is equal to the ſum of 25, 144, the ſquares of the two ſides 5, 12.

The firſt right angled triangle, having 1, 2, for its generating numbers is ſuch, that the difference of the two ſides 3, 4, is 1; and if you want to *find another of the ſame quality*, take 2 the greateſt of theſe generating numbers for the leaſt of the two in the triangle demanded; and in order to find the greateſt for this ſecond triangle, add 1, the leaſt of the firſt, to 4 the double of the greateſt of the firſt, and ſo you have 5 for your greateſt generating number of the ſecond right angled triangle, which conſequently is 20, 21, 29, where the difference of the two ſides 20, 21, is again 1.

If you deſire a third right-angled triangle of the ſame quality, make uſe of the laſt 20, 21, 29, after the ſame manner as you did the firſt, taking its greateſt generating number for the leaſt of the third, and adding its leaſt to the double of the greateſt, for the greateſt of this your third triangle; and ſo obſerving the ſame method, you may find a fourth, fifth, &c. as appears by this table.

Sides.		Hypoth.	Generat. numb.	
3	4	5	1	2
20	21	29	2	5
119	120	196	5	12
696	697	1025	12	29
4059	4060	5741	29	70
23660	23661	33461	70	169

The firſt right angled triangle 3, 4, 5, has likewiſe this quality, that the exceſs of the Hypothenuſe 5 above the ſide 4, is alſo 1, foraſmuch as the difference of the two generating numbers is 1, and for this reaſon you may *find an infinite number of other right-angled triangles of this quality,* if for their generating you take two that differ only by unity, as you ſee in this table.

Baſes.	Altitude.	Hypoth.	Generat.	numb.
3	4	5	1	2
5	12	13	2	3
7	24	25	3	4
9	40	41	4	5
11	60	61	5	6
14	84	85	6	7

Here you ſee the firſt differences of the baſes, 3, 5, 7, 9, *&c.* are equal, and the ſecond differences of the altitudes, 4, 12, 24, 40, *&c.* are likewiſe equal; and the ſame is the caſe of the Hypothenuſes, 5, 13, 25, *&c.*

Here the Baſes are odd numbers, and if you *would have them the ſquares of theſe odd numbers*, only take the altitudes and Hypothenuſes for the generating numbers of the triangles you propoſe, which by conſequence will run thus:

Baſes.	Heights.	Hypoth.	Gen.	numb.
9	40	41	4	5
25	312	113	12	13
49	1200	1201	24	25
81	3280	3281	40	41
121	7320	7321	60	61
169	14280	14281	84	85

If inſtead of one ſide you would have the *Hypothenuſe to be the ſquare number*, then your generating numbers muſt be the ſides of a right-angled triangle, as in the following ſcheme, where you ſee the Hypothenuſe is the ſquare of the greateſt generating number, with the addition of 1.

Sides.		Hypoth.	Gen.	numb.
7	24	25	3	4
119	120	169	5	12
336	527	625	7	24
720	1519	1681	9	40
1320	3479	3721	11	60
2184	6887	7225	13	84

The right-angled triangle, 21, 28, 35, has this quality, that the two ſides 21, 28, are triangular numbers, the ſides of which, 6 and 7, differ only by unity, and the ſquare (1225) of the Hypothenuſe (35) is likewiſe a triangular number, the ſide of which is 49.

The ſame is the quality of the triangle 820, 861, 1189, as alſo of the triangle 28441, 28680 40391, and of others.

The following right-angled triangles, which may be continued *in infinitum*, are ſuch, that their baſes and hypothenuſes are triangular numbers, and their heights cubic numbers.

Baſes.	Heights.	Hypoth.	Gen.	numb.
6	8	10	1	3
36	27	45	3	6
120	64	136	6	10
300	125	135	10	15
630	216	666	15	21
1176	343	1225	21	28

You may *find as many ſuch triangles as you will*, by adding and ſubtracting a ſquare number from its ſquare, for in the addition half the ſum is the hypothenuſe, and in ſubtracting half the remainder is the baſe, the height being equal to the cube of the root of the firſt ſquare number: or, which is the ſame thing, by taking for the generating numbers the triangular numbers in order, as you ſee in the ſcheme before us, where the leaſt generating numbers of one right-angled triangle is the greateſt of the preceding triangle.

SPORT XII.

Of Arithmetical Progreſſion.

BY Arithmetical Progreſſion, we mean a ſeries of quantities called Terms, that riſe continually by an equal exceſs, as 1, 3, 5, 7, 9, 11, *&c.* where the exceſs is 2; or 1, 4, 7, 10, 13, 16, *&c.* where the exceſs is 3; or 2, 6, 10, 14, 18, 22, *&c.* where they riſe by 4 at a time. And ſo of the reſt.

The principal property of arithmetical progreſſion, is this: Take three continnal terms, as 6, 10, 14, the ſum (20) of the two extremes (6, 14) is equal to the double of the middle term (10). Take four continual terms, as 6, 10, 14, 18, the ſum (24) of the two extremes (6, 18) is equal to that of the two middle terms (10, 14). In fine, in a larger number of continual terms, ſix for inſtance, as 2, 6, 10, 14, 18, 22, the ſum (24) of the two extremes (2, 22) is equal to that of any two terms that lie at an equal diſtance from them, as 6, 18, and 10, 14. From whence it is eaſy to conclude, that when a multitude of progreſſive terms is an odd number, the ſum of the extremes, or of thoſe equally remote, is the double of the middle term, as in theſe five terms, 2, 6, 10, 14, 18; for the ſum (20) of the extremes 2, 10, 18, or of the two equally remote, 6, 14, is the double of the middle term, 10.

You may readily find ſuch numbers as have this quality, that the ſum of their ſquares make a ſquare number, or, which is the ſame thing, the ſides of a right-angled triangle in numbers; and that by virtue of this double arithmetical progreſſion, $1\frac{1}{2}$, $2\frac{2}{5}$, $3\frac{3}{7}$, $4\frac{4}{9}$, where the exceſs is 2 in fractions, and 1 in whole numbers; for if you reduce the integer with its fraction to a fraction only, as $1\frac{1}{3}$ to $4\frac{4}{3}$, the numerator 4 and the denominator 3 will be the ſides of the right-angled triangle 3, 4, 5; and in like manner if you reduce $2\frac{2}{5}$ to $1\frac{2}{5}$, (which is done by multiplying the whole number 2 by the denominator 5, and adding to the product 10 the numerator 2) the denominator 5, and the numerator 12, will be the ſides of the right-angled triangle 5, 12, 13; and ſo of the reſt. Here you may ſee any odd number may be one of the ſides of a right angled triangle in whole numbers.

Inſtead of the double arithmetical progreſſion, you may make uſe of this, $1\frac{7}{8}$, $2\frac{11}{12}$, $3\frac{15}{16}$, $4\frac{19}{20}$,

$4\frac{19}{20}$, $5\frac{23}{24}$, *&c.* where the exceſs is 4 in fractions, and 1 in whole numbers, for if you reduce $1\frac{7}{8}$ to ^{15}, the denominator 8, and the numerator 15, will be the two ſides of the right-angled triangle 8, 15, 17; and in like manner if you reduce $2\frac{11}{12}$ to $\frac{35}{12}$, the denominator 12 and the numerator 35 will be the two ſides of another triangle 12, 35, 37; and ſo on. Here you ſee any odd number may be one of the ſides of a right-angled triangle in whole numbers.

In an arithmetical progreſſion, the ſum of the terms is equal to the ſum of the two extremes, multiplied by half the number of all the terms. And for this reaſon, in order *to find the ſum of any number of terms in Arithmetical Progreſſion*, for example, the ſum of theſe eight, 3, 5, 7, 9, 11, 13, 15, 17, you muſt multiply the ſum (20) of the two extremes (3, 17) by the number of the multitude of the terms (8) for then half the product (80 the half of 160) is the ſum you inquire for.

If on the other hand you know the ſum of the terms, the firſt term itſelf, and the number or multitude of the terms, you may find out what the terms are, by tracing the exceſs in this manner. Suppoſe the given ſum of the terms to be 80, the number of them 8, and the firſt term given 3, divide (160) the double of the ſum given (80) by the number given (8) then ſubtract from 20 the quotient, 6 the double of the firſt term given 3, and at laſt divide the remainder 14 by the given number wanting 1, that is 7, and the quotient 2 is the exceſs you look for, which added to the firſt term gives you 5 for the ſecond, and added to the ſecond 7 for the third, and ſo on.

If the ſum of the terms, their number, and the exceſs be given, we find out the firſt term, and by conſequence all the reſt after the manner of the third queſtion enſuing.

Queſtion I. *A gentleman bargains with a bricklayer to have a well ſunk upon theſe terms; he is to allow him three livres for the firſt toiſe (a toiſe is 6 foot) of depth, 5 for the ſecond, 7 for the third, and ſo on, riſing two livres every toiſe till the well is twenty toiſes deep.* Query, *how much will be due to the bricklayer, when he has digged twenty toiſes deep?*

To reſolve this queſtion, multiply the 2 livres augmentation money at every toiſe, by the number of the toiſes, bating 1, that is by 19, to the product 38 add 6 the double of 3 the number of livres promiſed for the firſt toiſe, then multiply the ſum 44 by half the number of all the toiſes, *viz.* 10, and the product ſhews you 444 livres due to the bricklayer for ſinking the well 20 toiſes deep.

Queſt. II. *A gentleman travelled* 100 *leagues in eight days, and every day travelled equally farther than the preceding day. Now it being diſcovered that the firſt day he travelled two leagues, the queſtion is how many days he travelled on each of the other days?*

To reſolve this queſtion, divide 200, the double of the leagues given 100, by 8 the number of days given, and from the quotient 15, ſubtract 4, the double of 2 the given number of leagues that he travelled the firſt day. Divide the remainder 21 by 7, the given number of days wanting one, and the quotient 3 ſhews that he travelled every day three leagues more than the day before, from whence it is eaſy to conclude, that ſince he travelled two leagues the firſt day, he travelled 5 the ſecond, 8 the third, and ſo on.

Queſt. III. *A traveller went* 100 *leagues in* 8 *days, and every day three leagues more than the preceding day. 'Tis aſked how many leagues he travelled a day?*

Divide 200, the double of the leagues given 100, by 8 the number of days given, and from the quotient 25 ſubtract 21, the product of 3 the number of the daily increaſe multiplied by 7 the given number of days bating 1. The remainder being 4 half it, and that ſhews you he travelled 2 leagues the firſt day; from whence it is eaſy to gather that he travelled 5 the ſecond, 8 the third, and ſo on.

Queſt. IV. *A robber being purſued, travelled* 8 *leagues a day; an Archer, who was the purſuer, made but* 3 *leagues the firſt day,* 5 *the ſecond,* 7 *the third, and ſo on, increaſing* 2 *leagues every day. The queſtion is in how many days the archer will come up with the robber, and how many leagues they will have travelled?*

To reſolve this and ſuch like queſtions, add 2 the number of the daily increaſe of leagues by the archer, to 16 the double of 8 the number of leagues made every day by the robber: from the ſum 18 ſubtract 6 the duplicate of 3, the number of leagues that the archer travelled the firſt day. The remainder 12, divide by 2 the number of the archer's daily increaſe; and the quotient 6 will ſhew you, that the archer will come up with the robber at the end of 6 days, and conſequently both of them muſt by that time have travelled 48 leagues, for 6 times 8 is 48, and the ſame is the ſum of theſe ſix terms of Arithmetical Progreſſion, 3, 5, 7, 9, 11, 13.

Queſt. V. *A great ſhip purſues a little one, ſteering the ſame way, at the diſtance of four leagues from it, and ſails twice as faſt as the ſmall ſhip. It is, aſked how far the great ſhip muſt ſail before it overtakes the leſſer?*

The diſtance of the two ſhips being 4, and their celerities being in a double Ratio, continue *in infinitum*, the double geometrical progreſſion, 4, 2, 1, $\frac{1}{4}\frac{1}{2}\frac{1}{8}$, *&c*, the firſt and the greateſt terms of which is 4; and find the ſum of all the infinite terms, by dividing 16 the ſquare of the firſt 4, by 2 the difference of the two firſt, and the quotient 8 directs that the great ſhip muſt make 8 leagues before ſhe can come up with the other.

SPORT XIII.

Of Magical Squares.

BY a Magical Square, we underſtand a ſquare divided into ſeveral other ſmall equal ſquares, filled with terms of an arithmetical progreſſion, ſo tranſpoſed, that all of the ſame line or rank, whether longitudinal, tranſverſe, or diagonal, make the ſame ſum.

This is the ſquare here annexed, divided into 25 little boxes or ſquares, in which the firſt 25 natural numbers are ſo tranſpoſed, that the ſum of each rank from above downward, or from the right to the left, or along the diagonals or diameters of the ſquares, is every way 65; which ſum 65 is in all an odd ſquare number, that is, it contains an odd ſquare number of places, *viz.* 25, and is equal to the product ariſing from 5, the root of the ſquare number 25; multiplied by 13, the middle term of the arithmetical progreſſion, 1, 2, 3, 4, *&c.*

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

This ſum is likewiſe found, by diſpoſing the given terms of the arithmetical progreſſion, according to their natural ſeries, 1, 2, 3, 4, &c. in the ſquare places, as you ſee here; for then the ſum of each diagonal rank that is the rank extending from one corner of the ſquare to the other, is the ſum demanded. This will likewiſe hold in even ſquares, or thoſe which contain an even ſquare number of boxes.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

In order to diſpoſe magically in the boxes of an odd ſquare: for inſtance, that of 25 boxes, having 5 for its ſide; to diſpoſe, I ſay, as many given numbers in arithmetical progreſſion, as 1, 2, 3, 4, 5, and ſo on till you come to the laſt and greateſt 25; write the firſt and the leaſt immediately under the middle box, or that which poſſeſſes the center of the ſquare; and moving diagonal wiſe to the right, write the ſecond term 2 in the adjacent box, the lowermoſt of the next right hand rank. Here proceeding in the courſe of the diagonal from left to right you find no place for number 3, and ſo are to place it in the oppoſite or uppermoſt box of the rank into which it ſhould have fallen. In like manner, finding no place for 4, you are to place it in the oppoſite box of the rank that it falls to on the outſide.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Thus you continue proceeding ſtill diagonal wiſe to the right; but in regard 6 falls to a place that is already filled with 1, you muſt there take a retrogade diagonal courſe from the right to the left, and write 6 in the lowermoſt ſtation of the rank in which the foregoing term 5 was placed, and ſo there will remain an empty place between 5 and 6. This retrogade courſe muſt always be obſerved when you fall in with a ſtation already poſſeſſed. Continue to place the reſt in order, according to theſe rules till you come to the angle of the ſquare, where in this example ſtands 15: then, foraſmuch as you can no longer move diagonal-wiſe to the right, you muſt place the term 16 in the ſecond place (from the top) of the ſame rank; this done, the reſt may be placed as the former, without any difficulty.

There are ſeveral magical diſpoſitions both for odd and even ſquares; but theſe being difficult to underſtand, we reckon them improper for Arithmetical Recreations.

This ſquare was called Magical, from its being in great veneration among the Egyptians, and the Pythagoreans their diſciples, who to add more efficacy and virtue to this ſquare, dedicated it to the ſeven planets divers ways, and engraved it upon a plate of the metal that ſympathized with the planet. The ſquare thus dedicated, was incloſed with a regular Polygon, inſcribed in a circle divided into as many equal parts as there were units in the ſide of the ſquare: with the names of the angles of the planet; and the ſigns of the zodiac written upon the void ſpaces between the Polygon and the circumference of the circle-circumſcribed. Through vain ſuperſtition they believed that ſuch a medal or Taliſman would befriend the perſon that carried it about him upon occaſion.

They attributed to Saturn the ſquare of 9 places or boxes, 3 being the ſide, and 15 the ſum of numbers in each row or column; to Jupiter the ſquare of 16 places, 4 being the ſide, and 34 the ſum of the numbers in each row; to Mars the ſquare with 25, 5 being the ſide, and 65 the ſum of numbers in each rank; to the Sun the ſquare with 36, 6 being the ſide, and 111 the ſum of each row; to Venus that of 49, 7 being the ſide, and 175 the ſum of numbers in each rank or column; to Mercury that of 64, 8 being the ſide, and 260 the ſum of each column; to the Moon the ſquare with 81 lodges, having 9 for its ſide, and 369 for the ſum of each column.

In fine, they attributed to imperfect matter, the ſquare with 4 diviſions, having 2 for the ſide; and to GOD the ſquare of only 1 lodge, the ſide of which is an unit, which multiplied by itſelf, undergoes no change. By virtue o this Sport, we are taught to reſolve the following Queſtion.

Queſtion. *To draw up in three ranks the nine firſt cards, from an ace to a nine, in ſuch a manner that all the points of each rank, taken either lengthwiſe or breadthwiſe, or diagonalwiſe, may make the ſame ſum.*

4	9	2
3	5	7
8	1	6

Diſpoſe the nine firſt natural numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, magically, according to the directions laid down above, and as you ſee it done here, and place the cards according to their number, anſwerable to theſe figures.

8	256	2
4	16	64
128	1	32

Inſtead of an arithmetical progreſſion, you may take a geometrical; for inſtance, this double progreſſion, 1, 2, 4, 8, 16, 32, 64, 128, 256, &c. and placing them magically, as above, you will find the product of each rank will be equal, *viz.* 4096, which is juſt the cube of the middle term 16.

1260	840	620
504	420	360
310	280	252

Here we ſhall add by the by, one ſquare more of 9 ſtations, in which the numbers of each rank taken any away, as above, are in harmonical proportion; and you may find as many other numbers of the ſame quality, as you will, if inſtead of the foregoing numbers you put letters, as you ſee it done underneath, where the literal magnitudes of each rank are harmonically proportional; and ſo by giving different value to the three undetermined letters, *a*, *b*, *c*, you will have inſtead of literal quantities, numbers that will always preſerve an harmonic proportion in each rank.

$$\begin{array}{ccc}
a & \dfrac{2ac}{a+c} & c \\[2ex]
\dfrac{2cb}{a+b} & \dfrac{2bc}{b+c} & \dfrac{2abc}{2ab+ab-ba} \\[2ex]
b & \dfrac{2abc}{2ac+ab-bc} & \dfrac{abc}{ab+ac-bc}
\end{array}$$

3	10	25	16	11
20	8	19	12	6
5	9	13	17	21
22	14	7	18	4
15	24	1	2	23

12	25	6	19	3
5	11	24	8	17
16	4	13	22	10
9	18	2	15	21
23	7	20	1	14

1	20	23	16	5
4	18	9	12	22
15	7	13	19	11
24	14	17	8	2
21	6	3	10	25

11	24	17	10	3
4	12	25	18	6
7	5	13	21	19
20	8	1	14	22
23	16	9	2	15

11	22	9	20	3
2	14	25	8	16
19	5	13	21	7
10	18	1	12	24
23	6	17	4	15

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

SPORT XIV.

Of Arithmetical Triangle.

BY an Arithmetical Triangle, we mean the half of a ſquare like a magical ſquare, divided into ſeveral ſmall and equal ſtations or points which contain the natural numbers 1, 2, 3, 4, &c. the triangular numbers 1, 3, 6, 10, &c. which are formed by the continual addition of

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	
1	3	6	10	15	21	28		
1	4	10	20	35	56			
1	5	15	35	70				
1	6	21	56					
1	7	28						
1	8							
1								

the foregoing numbers; the Pyramidal numbers 1, 4, 10, 20, &c. formed by the continual addition of the triangular; the Pyramido-Pyramidals 1, 5, 15, 35, &c. formed by the continual addition of the Pyramidal; and ſo on, as you ſee in the following cut.

Among the different uſes of the arithmetical triangle, I ſhall only ſingle out thoſe relating to Combinations, Permutations, and the Rules of Game; the reſt being too ſpeculative for Arithmetical Recreations.

By Combinations we underſtand all the different choices that can be made of ſeveral things, the multitude of which is known, by taking them divers ways, one by one, and two, three and three, &c. without ever taking the ſame twice.

For example, if you have four things expreſſed, by theſe four letters, *a*, *b*, *c*, *d*; all the different ways of joining two of them, as *ab*, *ac*, *ad*, *bc*, *bd*, *cd*; or three of them, as *abc*, *abd*, *acd*, *bad*; theſe I ſay, are called Combinations. And from hence it is eaſy to apprehend, that when four things are propoſed, you may take them one by one four ways; two and two ſix ways; three and three four ways; and by fours only one way; ſo that 1 in 4 combines four times; 2 ſix times; 3 four times; and 4 only once.

To find in a greater number of different things, ſuch as ſeven; the divers combinations that may be made by taking them divers ways, whether by Addition or Multiplication: as, if you would know all the poſſible conjunctions of the ſeven planets, taking them two by two; that is to ſay, if you would know how often 2 combines in 7; add an unite to each of the two numbers given, 2, 7, and ſo you have 3, 8, which gives us to know, that in the third ſtation (reckoning from below upwards, or from above downwards) of the eighth diagonal of the arithmetical triangle, you'll have the number of combinations demanded, *viz.* 21.

Or elſe, the two numbers given being 2 and 7, add together all the numbers of the ſecond rank, till you come at the ſeventh diagonal, *viz.* 1, 2, 3, 4, 5, 6, and the ſum 21 is what you want.

When the number of things propoſed goes beyond 9, the triangle here delineated can't ſerve you; and therefore we ſhall give this general rule for any number whatſoever.

The two numbers given being 2 and 7, to know how often 2 the leaſt will combine in 7 the greateſt; make of them theſe two arithmetical progreſſions 2, 1, and 7, 6, which decreaſe by an unit, and ought to have but two terms, that is, as many as the leaſt number 2 has units. Then multiply together all the terms of each progreſſion, that is, 7 by 6, and 2 by 1; and divide the firſt product 42 by the ſecond 2, and the quotient 21 ſatisfies the demand.

By this, or the foregoing method, you'll diſcover that three combines in 7, 35 times; 4 likewiſe 35 times; 5, 22 times; and 6 only 7 times. Whence it follows, that the number of all the combinations poſſible of ſeven different things, taken one by one, by two's, by threes, by fours, by fives, by ſixes and ſevens, amounts to 127, as appears by the addition of all the particular combinations, 7, 21, 35, 35, 21, 7, 1, which anſwer the numbers, 1, 2, 3, 4, 5, 6, 7. But you may find this total yet eaſier, by forming this double geometrical progreſſion, 1, 2, 4, 8, 16, 32, 64, conſiſting of ſeven terms, anſwerable to the number of things combined, *viz.* 7; for the ſum of theſe terms, 127, is the number you look for; which may ſtill be found yet an eaſier way, *viz.* ſubtract 1 from the propoſed number of things 7, and the remainder 6, directs you to take the ſixth power (64) of the number 2; and the double of that power, bating an unit, 127, is the number deſired.

Before I diſmiſs this ſubject, I ſhall here ſet down two methods peculiar to 2 and 3, for finding out how often theſe two numbers may be combined in any number of things. Suppoſe the number of things given is 7, you'll find how often 2 will combine in it, by ſubtracting the given number 7 from its ſquare 49; and taking (21) the half of the remainder, 42, for the number deſired. You'll find how often 3 may combine in 7, by adding 14, the double of 7 to 343, the cube of the ſame given number 7, and ſubtracting from the ſum (357) the triple (147) of the ſquare (49) of the ſame number (7) for then the ſixth part (35) of the remainder (210) ſhews you, that three will combine in 7 35 times.

There

Maria Mowbray

There is another ſort of combinations, that may be called Permutation, in which we take the ſame thing twice; as if you would combine theſe three numbers by two's, 2, 5, 6, in order to know what different quantities they can produce, if you conſider the two firſt thus, 25, you'll call them twenty-five; if thus, 52, you'll call them fifty-two; in like manner, the firſt and third taken thus, 26, is a quite different quantity from the ſame two taken thus, 62; and ſo of all others. From whence it appears, that the multitude or number of permutations is the double of that of combinations.

Permutations are of very good uſe in making anagrams, and ſometimes give very lucky hints; as in the word *ROMA*, the letters of which being tranſpoſed make this other word *AMOR*; but it is a much luckier hit that we meet with in theſe two Latin verſes;

Signa te, ſigna, temere me tangis & angis,
Roma tibi ſubito motibus ibit amor.

the letters of which being read backwards, form the ſame verſes.

We likewiſe make uſe of Permutations in playing at dice, to know the number of chances that attend the engaging to throw with two dice, 9 for inſtance; it being certain that the perſon who engages has four chances for it; for 9 may come up four ways, by *quatre cinque*, by *cinque quatre*, by *tres ſix*, and again by *ſix* *tres* (according as the firſt or ſecond dye happens to appear).

To give the joint combinations of ſeveral letters, for example theſe four *AMOR*, that is to find the number of their ſimple permutations, by tranſpoſing them all poſſible ways; make this arithmetical progreſſion, conſiſting of as many terms as there are letters to combine together, which in this example are four; ſo that the firſt term is always an unit, and the laſt denotes the number of letters; then multiply together all the terms, and the product 24 is the number of permutations or different changes that theſe four letters *AMOR* can undergo, as you ſee here;

AMOR
AMRO
AOMR
AORM
ARMO
AROM

MARO
MAOR
MOAR
MORA
MRAO
MROA

OAMR
OARM
OMAR
OMRA
ORAM
ORMA

ROMA
ROAM
RMAO
RMOA
RAMO
RAOM

By the ſame way do we find the number of Permutations of any other number of letters, *viz.* By making a progreſſion of as many natural numbers as there are letters to combine, and multiplying together all the terms of the progreſſion. Thus you will find that five letters may be tranſpoſed 120 ways; ſix 720; and ſo on as in the following table, where you ſee the 23 letters of the alphabet may be combined 25852016738884976640000.

1 | 1. A.

1	1.	A.
2	2.	B.
3	6.	C.
4	24.	D.
5	120.	E.
6	720	F.
7	5040.	G.
8	40320.	H.
9	362880.	I.
10	3628800.	K.
11	39916800.	L.
12	479001600.	M.
13	6227020800.	N.
14	87178291200.	O.
15	1307674368000.	P.
16	20922789888000	Q.
17	355687428096000.	R.
18	6402373705728000.	S.
19	121645100408832000.	T.
20	2432902008176640000.	V.
21	51090942171709440000.	X.
22	1124000727777607680000.	Y.
23	25852016738884976640000.	Z.
24	620448401733239439360000.	
25	15511210043330985984000000.	

Maria Mowbray her
Book Dated this 24th
Day of April 1819 —
if it be lost or Stole
I pray the finder any
Send it home

This table is eaſily calculated; for having diſcovered that four letters, for example, may be combined or tranſpoſed 24 ways; for if you multiply 24, the number of combinations, by 5 the next number, you have 120 for the combinations of five letters; and that multiplied by the next number 6, makes 720 for the combinations of ſix letters; and ſo on through all the ſucceeding letters.

By Parti, in the way of gaming, we underſtand the juſt diſtribution or adjuſtment of what money out of the ſtakes belonging to ſeveral players, who play for it ſo many games, or a certain number of parts or ſets, in proportion to what every one has ground to hope from fortune, upon the ſets he wants to be up.

For example, If two gameſters have ſtaked down forty piſtoles, which is then no longer their property only by way of retaliation, they have a right to what chance may bring them, upon the conditions ſtipulated at the firſt agreement; ſuppoſe they were to play for theſe 80 piſtoles three ſets, that the firſt had gained one ſet, and the ſecond none; that is, the firſt wants two ſets to be out and the ſecond three; theſe ſuppoſitions being laid down, and the gameſters having a mind to draw their ſtakes, without ſtanding to their chances, the juſt Quota appertaining to each, is what is called Parti, and is found out by the arithmetical triangle, after this manner:

Since the ſuppoſition runs, that the firſt gameſter wants 2 ſets, and the other three, and the ſum of the two numbers 2 an 3 is 5; we muſt turn to the fifth diagonal of the arithmetical triangle, and there take five the ſum of the two firſt numbers 1, 4, by reaſon of the two ſets that the firſt gameſter is ſhort; and 11 the ſum of the other three, 6, 4, 1, by reaſon of the three ſets that the ſecond gameſter is ſhort: and theſe two ſums 5 and 11 give the reciprocal Ratio of the two Parti's inquired for; ſo that the Parti or Quota of the one or firſt is to that of the ſecond, as 11 to 5.

But to adjuſt theſe Quota's, that is, to aſſign each gameſter his poſitive ſhare of the 80 piſtoles at ſtake, this number 80 muſt be divided into two parts proportional to the two terms 11, 5; and this is done by multiplying 80 by the two ſums 11, 5, ſeparately, and dividing each of the two products, (880, 400) by 16, the ſum of the two terms 11, 5; by which means you have 55 for the number of piſtoles due to the firſt gameſter that gained a ſet; and 25 for the other that gained none.

In like manner, if the firſt wants but 1 ſet to be out, and the ſecond 2, we add together theſe two numbers 1, 2, and their ſum being 3, turn to the third diagonal of the arithmetical triangle, and there take the firſt number 1, and the ſum 3 of the two others 2, 1; from theſe two numbers 1, 3, we learn that the firſt his Quota is to that of the ſecond as 3 to 1; and ſince the ſum of theſe two terms is 4, the conſequence is, that the firſt gameſter ought to have $\frac{3}{4}$ of the 80 piſtoles ſtaked, and the ſecond only $\frac{1}{4}$, that is, the firſt 60 piſtoles, and the other 20.

Hence it appears, that when the game is at this paſs, the firſt may lay upon the ſquare 3 to 1: and this we can likewiſe make out without the arithmetical triangle, after the following manner:

Since the firſt wants one ſet to be out, and the ſecond two, we muſt conſider, that if they went on with the game, and the ſecond gained a ſet, then the two gameſters would have equal chances, and ſo their quota's or dividends would be equal, it being a conſtant and a general rule that the one ſhare of the firſt is to that of the ſecond, as the chances of the one are to thoſe of the other. And ſo in this ſuppoſition, each of them has a title to an equal half of the money. 'Tis therefore certain, that if the firſt gains the ſet that is to be played, he ſweeps all; but if he loſes it, he has a title to an equal half: and therefore, if they have a mind to draw without playing the ſet, the firſt ought to have half the money at ſtake, and the half of the remaining half, that is $\frac{3}{4}$ of the whole; ſo that $\frac{1}{4}$ remains to the ſecond, for it is evident, that if a gameſter has a right to a certain ſum, in caſe he gains, and to a leſſer in caſe he loſes, he has a right to the half of thoſe two taken together, if the game is thrown up.

PASTIMES

This firſt caſe directs us to the ſolution of the ſecond, which ſuppoſes the firſt to want one ſet to be out, and the ſecond three; for if the firſt gains the ſet, he ſweeps all the 80 piſtoles; if he loſes, it turns to the firſt caſe as above, that is, he has a right only to $\frac{3}{4}$; and therefore, if the ſtakes are drawn without playing that ſet, his right is half of theſe two ſums taken together, *i. e.* $\frac{7}{8}$ or 70 piſtoles, $\frac{1}{8}$ or 10 piſtoles remaining to the ſecond.

This leads us to a reſolution of a third caſe. Suppoſing the firſt to be two ſets ſhort, and the ſecond three; for if the firſt gains the next ſet, he has a right to $\frac{7}{8}$ of the money, by the ſecond caſe; if he loſes it, ſo that the ſecond caſe wants only two to be out, as well as he, the money is to be equally divided between them. Upon the whole, the game ſtands thus; if the firſt wins, he claims $\frac{7}{8}$, if he loſes, he claims $\frac{1}{2}$; and therefore, if the game is thrown up without playing this ſet, he claims the half of theſe two ſums put together, *i. e.* $\frac{11}{16}$ or 55 piſtoles, leaving $\frac{5}{16}$ or 25 to the ſecond.

The ſecond caſe leads us likewiſe to the ſolution of a fourth caſe, in which the firſt is ſuppoſed to be one ſet ſhort of the whole, and the ſecond four; for if the firſt gains a ſet, he carries the 80 piſtoles; if he loſes it, ſo that the ſecond lacks only three to be out, he claims $\frac{7}{8}$ by the ſecond caſe. Now ſince, in caſe of winning, he takes 80 piſtoles, and in caſe of loſing $\frac{7}{8}$ of them, his dividend, upon throwing up, is the half of theſe two ſums put together, that is, $\frac{15}{16}$, or 75 piſtoles, and ſo he leaves $\frac{1}{16}$, or 5 piſtoles for the ſecond.

The fourth and third caſes leads us after the ſame manner, to the ſolution of a fifth, which ſuppoſes that the firſt gameſter is two ſets ſhort, and the ſecond four; for if the firſt gains a ſet, and ſo lacks but one to be out, he claims $\frac{15}{16}$ by the fourth caſe; and if he loſes it, ſo that the ſeconds wants but three, he claims $\frac{11}{16}$ by the third; and conſequently, in caſe of drawing, his due is the half of theſe two ſums put together, that is, $\frac{13}{16}$, or 65 piſtoles, $\frac{3}{16}$, or 15 piſtoles being left for the ſecond. And ſo of the other caſes.

All theſe, and an infinite number of other caſes that may happen, are ſolvable without the arithmetical triangle, after a different and an eaſy manner, as follows:

Take the fifth caſe for inſtance, which ſuppoſes the firſt to be two ſets ſhort, and the ſecond four; in this ſuppoſition the two gameſters want between them ſix ſets to be out: Take 1 off the ſix, and ſince the remainder is 5, ſuppoſe theſe five letters of the ſame form *a a a a a*, to favour the firſt gameſter; and theſe five *b b b b b*, to favour the ſecond; make combinations of theſe ten letters, as you ſee it here done; where of 32 combinations, the firſt 26 to the left, having at leaſt two *a*, are taken for the number of chances that can make the firſt to win; becauſe he lacks two ſets, and the remaining 6 to the right, or where there are at leaſt four *b*, are taken for the number of chances upon which the ſecond may win; becauſe he wants four to be out.

aaaaa
aaaab
aaaba
aabaa
abaaa
baaaa

aaabb
aabba
abbaa
bbaaa
aabab
abaab
baaab
baaba
babaa
ababa

aabbb
abbba
bbbaa
ababb
abbab
ababb
baabb
babba
bbaba
babab

abbbb
abbba
babbb
bbabb
bbbab
bbbbb

Thus it is plain, that the firſt his due is to that of the ſecond as 26 to 6, or, as 13 to 3.

In like manner to ſolve the third caſe, which ſuppoſes the firſt to want two ſets to be up, and the ſecond three, ſo that they want five between them; take 1 from the ſaid ſum 5, and ſince the remainder is 4, ſuppoſe theſe ſimilar letters *a a a a* to be favourable to the firſt, and theſe four *b b b b* to the ſecond, and combine theſe eight letters together, as you ſee it here done; where, of the 16 combinations, the firſt 11 to the left, having at leaſt two *a*'s, muſt repreſent the number of chances that the firſt has for game, two ſets being what he wants; and the remaining 5 to the right, having at leaſt three *b*'s, muſt be taken for the number of chances that can make the ſecond up, he being three ſets ſhort. Thus the claim of the firſt is to that of the ſecond, as 11 to 5, *&c.*

aaaa
aaab
aaba
abaa
baaa

aabb
abba
bbaa
baab
baba
abab

abbb
bbba
bbab
babb
bbbb

The ſame 16 combinations will ſerve for the ſolution of the fourth caſe, in which the firſt was ſuppoſed to be one ſet ſhort, and the ſecond four; ſo that five is the number of ſets wanted between them, as in the third caſe. For among theſe 16 you will find 15 that have at leaſt one *a*, (anſwerable to the one ſet that the firſt wants) for the chances upon which the firſt will win; and only one that has four *b*'s, the ſecond being four ſets ſhort, which ſhews there is but one chance that can ſave the ſecond. Thus the firſt ſhare is to that of the ſecond, as 15 to 1. And ſo of all other caſes.

To know, when two are at play, what advantage one has, that engages to throw 6, for example, with one dye at a certain number of throws, and firſt of all, at the firſt throw; we muſt conſider that his caſe is 1 to 5; for he has but one chance to win, and five to loſe upon; and conſequently if he lays upon one throw, he ought to lay but 1 to 5.

To engage to throw 6 with one dye at two throws, is the ſame thing, as to throw two dyes at a time, one of which is to be a 6; and in that caſe, he who throws has but 11 chances to win upon, ſince he may throw the firſt 6, and the ſecond 1, 2, 3, 4, or 5; or the ſecond 6, and the firſt 1, 2, 3, 4, or 5; or elſe both dyes ſixes; whereas he has 25 to loſe upon, as you ſee here. Where it is eaſy to conclude, that he who offers to throw with one dye at two throws, ought to ſet but 11 to 25.

1 1	2 1	3 1	4 1	5 1
1 2	2 2	3 2	4 2	5 2
1 3	2 3	3 3	4 3	5 3
1 4	2 4	3 4	4 4	5 4
1 5	2 5	3 5	4 5	5 5

When you lay upon 6 at two throws, take notice that 36, the ſum of all the chances, 11, 25, is the ſquare of the given number 6; and that 25, the number of chances againſt him who throws, is the ſquare of the ſame number, wanting 1, that is, 5. And therefore to find the number of chances that favour him who is to throw, you need only take 1 from 12, the double of the number given, and the remainder 11 is the number required; which being ſubtracted from 36, the ſquare of the former number 6, leaves 25 the remainder, which will always be a ſquare number, and denote the chances againſt him.

To lay upon 6 at three throws with one dye is the ſame as to lay upon 6 at one throw with three dice; and in that caſe, he who throws has 91 favourable chances, and 125 againſt him, and ſo ought to ſet but 91 to 125; thus you ſee he is at a loſs who lays upon the ſquare for 6 at three throws of one dye.

Take notice that the ſum 216 of all the chances 91, 125, is the cube of the given number 6, when you engage to throw 6 at three throws with one dye; and that 125, the number of the chances againſt you is the cube of the ſame number given, leſs 1, *i. e* 5. And therefore, to find the number of chances that favour the perſon that throws, you need only to ſubtract 125, the cube of the given number 6, wanting 1 (*i. e.* 5) from 216 the cube of the ſame number given.

By the ſame method we find out what advantage he has who proffers to throw 6 with one dye at four throws; for if we ſubtract from the fourth power or biquadrate 1296 of the given number 6, if we ſubtract, I ſay, from that, 625 the biquadrate of the ſame number, leſs one, or of 5, the remainder ſhews us 671 favourable chances for him that throws; the biquadrate 625 being the number of the chances againſt him: So that he who lays upon 6 at four throws has the odds on his ſide.

But he has a much greater advantage upon 6 at five throws with one dye, as appears by ſubtracting 3125, the fifth power of 5 (the given number, bating 1) from 7776, the fifth power of the given number 6; for the remainder 4651, is the number of the favourable chances, and 3125, the fifth power ſubtracted, is the number of thoſe againſt him who throws.

If you want to know what advantage he has who offers with two or ſeveral dice, to throw at one throw a determined raffle; for example two *tres*; you muſt conſider, that with two dice he has but one chance to ſave him, and 35 to looſe upon, ſince two dice can combine 36 different ways, that is, their 6 faces may have 36 different poſtures, as you ſee by this ſcheme;

1	1	2	1	3	1	4	1	5	1	6	1
1	2	2	2	3	2	4	2	5	2	6	2
1	3	2	3	3	3	4	3	5	3	6	3
1	4	2	4	3	4	4	4	5	4	6	4
1	5	2	5	3	5	4	5	5	5	6	5
1	6	2	6	3	6	4	6	5	6	6	6

This number 36, is the ſquare of 6, the number of faces, there being but two dice; but if there were three, the cube of 6, 216, would be the number of combinations: and if there were four, the biquadrate of 6, 1296 would be the number. And ſo on.

From what has been ſaid, it is evident, that in engaging a determined raffle at one throw with two dice, one ought to lay but 1 to 35; and by a parity of reaſon, that he ought to lay 3 to 213 upon a determined raffle or pair-royal with three dice; and 6 to 1290 with four; for of the 216 chances of three dice, there's only three that can favour him, ſince three things can combine by two's only 3 ways; and of 1296 chances of four dice, only 6 can favour the thrower, ſince four things combine by two's 6 ways.

But if you want to know what odds he lies under who proffers to throw a raffle of one ſort or t'other at the firſt throw of two or more dice; you may find without difficulty, that he ought to ſet but 6 to 30, or 1 to 5 upon two dice, ſince of the 36 chances of two dice, there is only 6 that can make a raffle; and that upon three dice, his caſe is 18 to 198, or 1 to 11, ſince of the 216 chances, that three dice can fall upon only 18 can produce a raffle.

SPORT XV.

Several dice being thrown, to find the number of points that ariſe from them, after ſome operations.

SUPPOSE three dice thrown upon a table, which we ſhall call A, B, C, bid the perſon that threw them add together all the uppermoſt points, and likewiſe thoſe underneath of any two of the three: for inſtance, B and C; A being ſet apart, without altering its face. Then bid him throw again the ſame two dice, B and C, and make him add to the foregoing ſum all the points of the upper faces, and withal the lowermoſt points, or thoſe underneath of one of them, C for inſtance, B being ſet apart near A, without changing its face, for giving a ſecond ſum. In fine, order him once more to throw the laſt dye C, and bid him add to the foregoing ſecond ſum the upper points, for a third ſum, which is thus to be diſcovered. After the third dye C is ſet by the other two, without changing its poſture, do you come up, and compute all the points upon the faces of the three dice, and add to their ſum as many 7's as there are dice, that is, in this example 21, and the ſum of theſe is what you look for; for when a dye is well made, 7 is the number of the points of the oppoſite faces.

To exemplify the matter: Suppoſe the firſt thrown of the three dice, A, B, C, brought up 1, 4, 5; ſetting the firſt 1 apart, we add to theſe three points, 1, 4, 5, the points 3 and 2 that are found under or oppoſite to the upper points 4 and 5 of the other two dice; and this gives me the firſt ſum 15. Now ſuppoſe again that the two laſt dice are thrown, and ſhew uppermoſt the two points 3 and 6, we ſet that with the three point apart, near the die that had 1 before, and add to the foregoing ſum (15) theſe two points 3 and 6, and withal 1 the point that is found lowermoſt in the die that is ſtill kept in ſervice, and had 6 for its face at this throw; thus we have 25 for the ſecond ſum. We ſuppoſe at laſt, that this third and laſt die being thrown a third time, it comes up 6, which we add to the ſecond ſum 25, and ſo make the third ſum 31. And this ſum is to be found out by adding 21 to 10 the ſum of the points 1, 3, 6, that appear upon the faces or uppermoſt ſides of the three dice then ſet by.

SPORT XVI.

Two dice being thrown, to find the upper points of each die without ſeeing them.

MAKE any one throw two dice upon a table, and add 5 to the double of the upper points of one of them, and add to the ſum multiplied by 5, the number of the uppermoſt points of the other or the ſecond die; after that, having aſked him the joint ſum, throw out of it 25, the ſquare of the number 5 that you gave to him, and the remainder will be a number conſiſting of two figures; the firſt of which to the left repreſenting the tens, is the number of the upper points of the firſt die, and the ſecond figure to the right repreſenting units, is the number of the upper points of the ſecond die.

We will ſuppoſe that the number of the points of the firſt die that comes up is 2, and that of the ſecond 3; we add 5 to 4, the double of the points of the firſt, and multiply the ſum 9 by the ſame number 5, the product of which operation is 45, to which we add 3, the number of the upper points of the ſecond die, and ſo make it 48, then we throw out of it 25, the ſquare of the ſame number 5, and the remainder is 23, the firſt figure of which 2 repreſents the number of points of the firſt die, and the ſecond 3 the number of points of the ſecond die.

Another way of anſwering this Sport is this; aſk him who threw the dice, what the points underneath make together, and how much the under points of one ſurpaſs thoſe of the other; and if this exceſs is, for example, 1, and the ſum of all the lower points is 9, and theſe two numbers 1 and 9, and ſubtract the ſum 10 from 14; then take 2, the half of the remainder 4, for the number of the upper points of one of the dice; and as for the other die, inſtead of adding the exceſs 1, to the ſum 9, ſubtract it out of 9, and take the remainder 8 out of 14, 6 is the remainder, the half of which 3, is the number of the npper points of the ſecond die.

A third way is this: bid the perſon who threw the dice, add together the upper points, and tell you their ſum, which we here ſuppoſe to be 5; then give him orders to multiply the number of the upper points of one die by the number of upper points of the other die; and to acquaint you in like manner with their product, which we here ſuppoſe to be 6: now having this product 6, and the preceding ſum 5, ſquare 5, and from its ſquare 25 ſubtract 24, the quadruple of the product 6, and the remainder is 1: then take the ſquare root of the remainder, which in its caſe is 1, and by adding it to, and ſubſtracting it from the foregoing ſum 5, you have theſe two numbers 6, 4, the halfs of which, 3, 2, are the numbers of the upper points of each die.

SPORT XVII.

Upon the throw of three dice, to find the upper points of each die, without ſeeing them.

ORDER the perſon that has thrown the dice, to place them near one another in a ſtraight line, and aſk him the ſum of the lowermoſt points of the firſt and ſecond die, which we here ſuppoſe to be 9; then aſk him the ſum of the points underneath of the ſecond and third, which we here ſuppoſe to be 5; and at laſt the under points of the firſt and third, which we put 6. Now, having theſe numbers given you, 9, 5, 6, ſubtract the ſecond number 5 from 15, the ſum of the firſt and third, 9 and 6; and the remainder 10 from 14; ſo there remains 4, the half of which 2 is the number of the upper points of the firſt die. To find the number of the upper points of the ſecond, ſubtract the third number 6 from 14, the ſum of the two firſt 9 and 5; and the remainder 8 from 14 again; ſo you have a ſecond remainder 6, the half of which, 3, is the number demanded. At laſt for the third die, ſubtract the firſt number 9 from 11, the ſum of the ſecond and third 5, 6, and the remainder 2 from 14; ſo you have a ſecond remainder 12, the half of which, 6, is the number of the upper points of the third die.

SPORT XVIII.

To find a number thought of by another.

ORDER the perſon to take 1 from the number thought upon, and after doubling the remainder, to take 1 from it, and to add to the laſt remainder, the number thought upon. Then aſk him what that ſum is, and after adding 3 to it, take the third part of it for the number thought of. For example, let 5 be the number, take 1 from it, there remains 4; then take 1 from 8, the double of that 4, and the remainder is 7, which becomes 12, by the addition of 5, the number thought of; and that 12, by the addition of 3, makes 15, the third part of which, 5, is the number thought of.

Another way is this: After taking 1 from the number thought of, let the remainder be tripled; then let him take 1 from that triple, and add to the remainder the number thought of. At laſt, aſk him the number ariſing from that addition, and if you add 4 to it, you will find the fourth part of the ſum to be the number thought of. Thus 5, bating 1, makes 4, that tripled makes 12, which loſing 1 ſinks to

11, and enlarged by the acceſſion of 5, comes to 16, which, by the addition of 4, is 20, and the fourth part of that, *viz.* 5, is the number thought of.

3*d way.* Add 1 to the number thought of, double the ſum, and add 1 more to it, and then add to the whole ſum the number thought of. Having learned the ſum total, take 3 from it, and the third part of the remainder is what you look for. Thus, 5 and 1 is 6, and the double of that, enlarged by 1 is 13, which by the addition of 5, comes to 18; take 3 from that, the remainder is 15, the third part of which, 5, is the number thought of.

4*th.* Or elſe after adding 1 to the number thought of, bid the perſon triple the ſame, and add firſt 1 to it, and then the number thought of. At laſt, aſk the ſum of this laſt addition, and after robbing it of 4, take the fourth part of the remainder for the number thought of. Thus, 5 and 1 is 6, the triple of which and 1 is 19, which, with 5 is 24, and that bating 4 is 20, the fourth part of which, 5, anſwers the Sport.

5*th.* Take 1 from 5, the number thought of, double the remainder 4, from which, 8, take 1, and likewiſe the number thought of; after which, aſk for the remainder 2, and add 3 to it, ſo you have your number.

6*th.* Let the perſon that thinks add 1 to the five, the number thought of, and to the double of that, 12, 1 more, and ſubtract from the ſum 13, the number thought of; then aſk for the remainder 8, and taking 3 from it, what you leave behind, 5, is the number thought of.

7*th*. Bid the perſon that thinks take 1 from 5, the number thought of; and 1 from 12, the triple of the remainder; and then the double of the number thought of; 10 from 11, the laſt remainder. This done, aſk for the remainder of the third ſubtraction, *viz.* 1, and adding 4 to it, you will find ſatisfaction.

8*th*. Add 1 to the number thought of 5, adding 1 more to the triple of that you have, 19, from which take 10, the double of the number thought of; then aſk for the remainder, 9, from which take 4, and ſo you are right.

9*th*. Order the perſon to tripple the number thought of, (5) and out of the triple number (15) to caſt away the half, if it were poſſible; and ſince in this example it is not, to add 1 to it ſo as to make it 16; the half of which 8, muſt be trippled, and that makes 24. The perſon that thinks having done this, aſk him how many 9's are in the laſt triple (24); he anſwers two; ſo you are to take 2 for every 9, which in this example makes 4, and by reaſon of the 1 you gave to make the 15 an even number, you are here to repay it by addition to the 4, and ſo you have 5, the number thought of. If there happens to be no 9 in the laſt triple, the number thought of is 1.

10*th*. Bid him add 1 to the number thought of (which makes 6); then ſubtract it from it, and ſo it leaves (4) a remainder; then bid him multiply the ſum (6) into the remainder (4) and tell you the product. To this product 24 add 1, and of the ſum 25, take the ſquare root 5.

11*th*. Bid the perſon that thinks add 1 to the number thought of (which we all along ſuppoſe to be 5) and multiply the ſum (6) by the number thought of (5), then let him ſubtract the number thought of (5) from the product (30) and tell you the remainder (25) the ſquare root of which 5 is the number thought of.

12*th*. After taking 1 from the number thought of, bid him multiply the remainder (4) by the number thought of (5) and add to the product (20) the ſame number thought of, and tell you the ſum 25, of which you are to extract the ſquare root 5.

13*th*. Bid him add 2 to the number thought of, and clap a cypher to the right of the ſum, which makes 70; and to that add 12, to the ſum of which addition (82) let him clap another cypher, ſo as to make it 820. From this decuple (820) let him ſubtract 320, and tell you the remainder 500, from which you are to cut off the two cyphers (each of which did ſtill decuple the number it was put to) and ſo you have the number thought of 5.

14*th*. Let him add 5 to the double of the number thought of; to the ſum 15 let him add a cypher on the right-hand to decuple it; then let him add 20 to the ſum (150) and to the laſt ſum (170) ſet another decupling cyyher; at laſt let him ſubtract 700 from the laſt ſum, of all (1700)

(1700) and diſcover to you the remainder 1000, which from you are to ſtrike off two cyphers to the right, and take the half of the remainder (10) for the number thought of.

Theſe two laſt methods are not very ſubtile; for the laſt number being known, it is an eaſy matter, by a retrogade view, to find out the other numbers, and by conſequence the number thought of. And upon that conſideration we ſhall here ſubjoin two other methods that are more myſterious.

15*th*. Bid the perſon that thinks add 1 to the triple of the number thought of, and triple the ſum (16) again: to which laſt ſum (48) bid him add the number thought of (5); then aſk him the ſum of all (53) and from that take off 3, and the right-hand cypher from the remainder 50; which leaves you 5 to the left for the number thought of.

16*th*. Bid him take 1 from the triple of the number thought of (15) and multiply the remainder (14) by 3; and add to (42) the product, the number thought of (5); then aſk the ſum of the addition, 47, to which add 3, and cut off from the ſum 50 the cypher, which muſt needs be on the right-hand, and ſo leaves to the left the number thought of.

From theſe two laſt methods we may draw this inference, that *if we add an unit to the triple of any number* (as to 18 the triple of 6) *and the ſame number* (6) *to the triple of the ſum* (57) *the ſecond ſum* (63) *will always terminate with* 3.

Another inference is, that *if we ſubtract an unit from* [18] *the triple of any number* [6] *and add the ſame number* [6] *to the triple of the remainder* (51 the triple of 17) *the ſum* [57] *will always end with the figure* 7.

The laſt inference is, that this double Sport is impoſſible, viz. *To find a number of ſuch a quality, that if you add to, or ſubtract from its triple, an unit, and add the ſame number to the triple of the ſum of the remainder, the laſt ſum will be a perfect ſquare number*; for as we ſhewed in Sport 9 no number ending in 3 or 7 can be a true ſquare. See the following Sport.

SPORT XIX.

To find the number remaining after ſome operations, without aſking any queſtions.

LET another think of a number at pleaſure; bid him add to the double of it an even number, ſuch as you have a mind to. For example 8; then bid him ſubtract from half the ſum the number thought on, and what remains is the half of the even number that you ordered him to add before; and ſo you may roundly tell him you are ſure the remainder is 4.—— Though the demonſtration of this is eaſy, yet thoſe who are not appriſed of the reaſon will be ſurpriſed at it. However, that you may light exactly on the number thought of, conceal your knowledge of the remainder 4, and bid him ſubtract that remainder, whatever it is, from the number thought of, if ſo be it be larger; or elſe if the number be leſs, to ſubtract it from the remainder; and then aſk him for the remainder of the laſt ſubtraction; for if you add this remainder to the half of the even number you gave him (*i. e.* 4 the half of 8) when the number thought of is larger than that of the half of the even number; or if you ſubtract the remainder from the ſame half [4] when the number thought of is leſs than it, you will have the number thought of To exemplify the matter, let 5 be the number thought of, and 8 added to its double 10, which makes 18; the half of that is 9; and 5, the number thought of, ſubtracted from 9 leaves 4, the half of the additional number 8; and if you take this half 4 from the number thought of 5, there will remain 1, which being added to the ſame half 4 (the number thought of being greater than that half) gives 5, the number thought of. In like manner it to 10, the double of 5 the number thought of, you add 12, you will have 22, the half of which is 11; and from thence taking the number thought of 5, there remains 6, the half of the additional number 12: and if from that half 6, you take the number thought of 5, (which in this example is leſs than the ſaid half) there will remain 1, which being taken from the ſame half, ſince the number thought of is leſs than that half [6] leaves 5 for the number thought of.

But an eaſier way to anſwer the ſport is this: Bid the perſon that thinks, take from the double

ble of the number thought of, any even number you will that is leſs, for example 4; then let him take the half of the remainder from the number thought of, and what remains will be 2, the half of the firſt number ſubtracted 4; and therefore to find the number thought of, bid him add the number thought of to that half 2, and then aſk the ſum, 7, from which you are to take the ſame half, and ſo there will remain 5 for the number thought of.

But another, and yet eaſier way is this: Bid him add what number you will to the number thought of, and multiply the ſum by the number thought of; for if you make him ſubtract the ſquare of the number thought of from the product, and tell you the remainder, you have nothing to do but to divide that remainder by the number you gave him to add before; for the quotient is the number thought of. Thus 4 added to 5 (the number thought of) makes 9, which being multiplied by 5, makes 49; from which take 25, the ſquare of the number thought of, and there remains 20, which being divided by 4, leaves 5 in the quotient.

Or elſe bid the perſon that thinks, take a certain leſſer number from the number thought of, and multiply the remainder by the ſame number thought of; for if you make him take the ſquare of the number thought of from the product, and tell you the remainder; by dividing that remainder by the number you ordered to be taken from the number thought of, you have the number thought of in the quotient.

But of all the ways for finding out a number thought of, the following is certainly the eaſieſt; make him take from the number thought of what number you pitch upon that is leſs than it, and ſet the remainder apart; then make him add the ſame number to the number thought upon, and the preceding remainder to the ſum, for a ſecond ſum; which he is to diſcover to you, and the half of that ſum is the number thought of. Thus 5 being thought of, and 3 taken from it, the remainder is 2; and the ſame number 3 added to 5, makes 8, and that, with the preceding remainder, 10, the half of which, 5, is the number thought of.

SPORT XX.

To find the number thought of by another, without aſking any queſtions.

BID the other perſon add to the number thought of, its half if it be even, or its greateſt half if it be odd; and to that ſum its half or greateſt half, according as it is even or odd, for a ſecond ſum, from which bid him ſubtract the double of the number thought of, and take the half of the remainder, or its leaſt half, if the remainder be odd; and thus he is to continue to take half after half, till he comes to an unit. In the mean time you are to obſerve

ſerve how many ſubdiviſions he makes, retaining in your mind for the firſt diviſion 2, for the ſecond 4, for the third 8, and ſo on in a double proportion, remembring ſtill to add 1 every time he took the leaſt half; and that when he can make no ſubdiviſion, you are to retain only 8. By this means you have the number that he has halfed ſo often, and the quadruple of that number is the number thought of, if ſo be he was not obliged to take the greateſt half at the beginning, which can only happen when the number thought of is evenly even, or diviſible by 4; in other caſes, if the greateſt half was taken at the firſt diviſion, you muſt ſubtract 3 from that quadruple; if the greateſt half was taken only at the ſecond diviſion, you ſubtract but 2; and if he took the greateſt half at each of the two diviſions, you are to ſubtract 5 from the quadruple, and the remainder is the number thought of.

For example, let 4 be the number thought of, which by the addition of its half 2, becomes 6, and that, by the addition of its half 3, is 9: from which 8, the double of the number thought of, being ſubtracted, the remainder is 1, that admits of no diviſion; and for this reaſon you retain only 1 in your mind, the quadruple of which, 4, is the number thought of.

Again; let 7 be the number thought of; this being odd, the greateſt half of it, 4, added to it makes 11, which is odd again; and ſo the greateſt half of 11 added to 11, makes 17, from which we take 14, the double of the number thought of, and ſo the remainder is 3, the leaſt half of which is 1, that admits of no further diviſion. Here there being but one ſubdiviſion, we retain 2, and to that add 1 for the leaſt half taken, ſo we have 3, the quadruple of which is 12. But becauſe the greateſt moiety was taken both in the firſt and ſecond diviſion, we muſt ſubtract 5 from 12, and the remainder 7 is the number thought of.

SPORT XXI.

To find out two numbers thought of by any one.

HAVING bid the perſon that thinks add the two numbers thought of (for example 3 and 5) order him to multiply their ſum [8] by their difference [2] and to add to the product [16] the ſquare [9] of the leaſt of the two numbers [3] and tell you the ſum, 25, the ſquare root of which 5, is the greateſt of the two numbers thought of. Then for the leaſt, bid him ſubtract the firſt product [16] from the ſquare [25] of the greateſt number thought of [5] and tell you the remainder, 9, of which the ſquare root 3 is the leaſt number thought of.

An eaſier way of doing it is this: Bid him add to the ſum of the two put together [8] their difference [2] and tell you the laſt ſum, 10, for the half of it, 5, is the greateſt number thought of. And as for the leaſt, bid him ſubtract the difference of the two numbers thought of from their ſum, and aſk him the remainder, 6, 6, the half of which, 3, is the number you look for.

This Sport may likewiſe be ſolved after the following manner: Bid him ſquare the ſum of the two numbers *(which is 64 in this example)* then bid him add to the leaſt number thought of [3] the double [10] of the greateſt [5] and multiply the ſum [13] by the leaſt [3] and ſubtract the product [39] from the foregoing ſquare [64] and diſcover the remainder 25, the ſquare root of which is the greateſt number thought of; and as for the leaſt, order him to add to the greateſt [5] the double [6] of the leaſt [3], and multiply the ſum [11] by the greateſt [5] and ſubtract the product 55, from the foregoing ſquare [64] and tell you the remainder [9] the ſquare root of which is 3, the leaſt number thought of.

Another, and a very eaſy way, is this: bid him multiply the two numbers [5, 3] together; and then multiply the ſum of the two numbers [8] by the number you want to find, whether the greater or leſſer, and ſubtract the product of the two numbers [15] from that product (which is 40, if you want the greater, and 24, if you look for the leſſer number) and tell you the remainder, 25, or 9, the ſquare roots of which ſatisfies the demand.

Or elſe, bid him firſt take the product of the two numbers [15], then multiply their difference [2] by the number enquired for [3 or 5] and add to that product the product of the two numbers [15] if you want the greateſt, or ſubtract that product from the product of the two numbers, if you look ſor the leaſt. Then he telling you the ſum, or the remainder, their ſquare roots are the numbers in queſtion.

When the leaſt of the two numbers does not exceed 9, it is eaſy to find them out after this manner; let one be added to the tripple of the greateſt, and the two numbers thought of to the tripple of that ſum, and the total ſum diſcovered; from which you are to take off 3, and then the right-hand figure is the leaſt, and the left-hand figure the greateſt number thought of. Thus 5 and 3 being thought of, 1 added to the tripple of 5, is 16, and the tripple of that [48] added to 8, the ſum of the two numbers, makes 56, which loſing 3, is 53; 3 the right-hand figure being the leaſt, and 5 on the left the greateſt number thought of.

SPORT XXII.

To find ſeveral numbers thought on by another.

IF the quantity of numbers thought of is odd, aſk for the ſums of the firſt and ſecond, of the ſecond and third, of the third and fourth, and ſo on till you have the ſum of the firſt and laſt; and having written all theſe ſums in order, ſo that the laſt ſum is that of the firſt and laſt; ſubtract all the ſums of the even places from all thoſe in the odd places; and the half of the remainder is the firſt number thought of, which being ſubtracted from the firſt ſum, leaves the ſecond number remaining, and that ſubtracted from the ſecond, leaves the third number remaining, and ſo on to the laſt. For example, ſuppoſe theſe five numbers thought of, 2, 4, 5, 7, 8, the ſums of the firſt and ſecond, of the ſecond and third; and ſo on to the ſum of the firſt and laſt, and 6, 9, 12, 15, 10: and 24 the ſum of the even places, 9 and 15, being taken from 28, the ſum of the odd places, there remains 4, the half of which, 2, is the firſt number thought of, and that being taken from the firſt number 6, leaves 4 for the ſecond number, and 4 taken from the ſecond, 9, leaves 5 for the third, and ſo on.

If the quantity of numbers thought upon is even, aſk for the ſums of the firſt and ſecond, of the ſecond and third, of the third and fourth, and ſo on to the ſum of the ſecond and laſt; write them all in order, ſo that the ſum of the ſecond and laſt may be the laſt in order: take all the ſums in the odd places (excepting the firſt) from thoſe in the even, and the half of the remainder is the ſecond number thought of, and that taken from the firſt ſum, leaves the firſt number, which taken from the third ſum, leaves for a remainder the third number, and ſo on. Thus 2, 4, 5, 7, 8, 9, being the numbers thought of, the ſums propoſed, as above, are 6, 9, 12, 15, 17, 13. Then take 29 the ſum of 12 and 17 the odd places (excepting the firſt) out of 37 the ſum of 9, 15, 13, the three even ſtations, and the remainder is 8, the half of which, 4, is the ſecond number thought of;

and that taken from 6, the firſt ſum, leaves 2 the firſt number, as the ſame ſecond number 4, taken from the ſecond ſum 9, leaves 5 for the third number, which taken from the third ſum 12, leaves 7 for the fourth, and ſo on.

When each of the numbers thought of conſiſts only of one figure, they are eaſily found in the following manner: let the perſon add 1 to the double of the firſt number thought of, and multiply the ſum by 5, then add to the product the ſecond number thought of. If there is a third number, add 1 to the double of the preceding ſum, and after multiplying the whole by 5, add to the product the third number thought of. In like manner, if there is a fourth number, bid him add 1 to the double of the laſt preceding ſum, and after multiplying the whole by 5, add to the product the fourth number thought of, and ſo on, if there are more numbers. This done, aſk the ſum ariſing from the addition of the laſt number thought of, and ſubtract from it 5 for two, 55 for three, and 555 for four numbers thought of, and ſo on, if there are more; and then the firſt left-hand figure of the remainder is the firſt number thought of, the next (moving to the right) is the ſecond, the next to that the third, and ſo on till you come to the laſt right-hand figure, which is the laſt number thought of.

For example, let 3, 4, 6, 0, be the numbers thought of, and 1 added to 6, the double of the firſt 3, and the ſum 7 multiplied by 5, the product of which, 35, with the addition of the ſecond number 4, is 39; then 1 being added to 78, the double of 39, and the ſum 79 multiplied by 5, the product 395, with the addition of the third number 6, is 401; and the double of that, with the addition of an unit is 803, which multiplied by 5 is 4015, and with the addition of the fourth number, 9, 4024. Now, if from this ſum 4024, we take 555, the remainder is 3469, the four figures of which are the four numbers thought of.

But there is a method for this purpoſe that is ſtill eaſier, *viz.* Let 1 be ſubtracted from the double of the firſt number, and the remainder multiplied by 5, to the product of which multiplication, let the ſecond number thought of be added, then, if there be more numbers than two let him add 5 to the laſt ſum for a ſecond ſum; let 1 be taken from the double of this ſecond ſum, and the remainder multiplied by 5, and the third number added to that product; this done, if there are no more numbers thought of (otherwiſe you muſt add 5, and go on again) aſk for the laſt ſum, add 5 to it, and the figures of the whole ſum will repreſent the numbers thought of, as above.

For inſtance, let 3, 4, 6, 9, be thought of; take 1 from 6, the double of the firſt 3, multiply the remainder 5 by 5, add to the product 25, the ſecond number 4; to the ſum 29 add 5, which gives you 34 for a ſecond ſum, take 1 from 68, the double of this ſecond ſum, multiply the remainder 27 by 5, and to the product 335, add the third number 6, which makes 341; add 5 to this laſt ſum, then it makes 346, the double of which, wanting 1, is 691, and that multiplied by 5, 3455, which with the addition of the fourth number 9, is 3464. Now adding 5 to this ſum, you have 3469, the four figures of which repreſent the four numbers thought of.

SPORT XXIII.

A perſon has in one hand a certain even number of piſtoles, and in the other an odd number; it is required to find out in which hand is the even or the odd number?

LET the number in the right-hand be multiplied by any even number you will, as 2, and the number in the left by ſuch an uneven number as you pitch upon, as 3, then order the perſon to add together the two products, and take the half of their ſum, and if he can take an exact half, ſo that the ſum is even, you will know by that, that the number in the right-hand being multiplied by an even number is odd, and conſequently that in the left multiplied by an odd number is even. But on the contrary, if he cannot take an exact half, the number in the right is even, and that in the left odd.

For example, ſuppoſe 9 piſtoles in the right-hand, and 8 in the left; multiply 9 by 2, and 8 by 3; the ſum of the two products 42 being an even number, ſhews that 9, the odd number, multiplied by the even 2, is in the right-hand, and conſequently 8 the even in the left. This Sport directs us to the ſolution of the following queſtion:

Queſtion. *A man having a piece of gold in one hand and ſilver in the other, it is aſked in what hand the gold or ſilver is in?*

Fix a certain value in an even number, as 8, on the gold, and an odd, as 5, upon the ſilver. Direct the perſon to multiply the number anſwering to the right-hand by any even number, as 2, and that in the left by a determined odd number, as 3, and aſk him whether the joint ſum of the products is even or odd; or bid him half it, and ſo you will learn whether it is even or odd, without aſking. If this ſum is odd, the gold is in the right-hand; if even, *e contra.*

SPORT XXIV.

To find two numbers, the Ratio and difference of which is given.

TO find two numbers, the firſt of which, for example, is the ſecond as 5 to 2, and the difference or exceſs 12: multiply the difference 12 by 2, the *leaſt* term of the given Ratio, and divide the product 24 by 3, the difference of the two terms 5, 2, and you will find the quotient 8, the leaſt of the two numbers looked for, and that added to the difference 12, *viz.* 20, the greateſt.

If you will, you may multiply the given difference by the *greateſt* term of the given Ratio, and after dividing the product by the difference of the two terms of the Ratio, you will find the quotient the great number, which, upon the ſubtraction of 12, leaves the leſſer remaining. Or you may take this way; multiply each of the two terms of the given Ratio, by the difference given, and divide each of the products by the difference of the two terms, and the quotients are the numbers demanded. This Sport furniſhes an eaſy ſolution to the following queſtion:

Queſtion. *If a man has as many pieces of money in one hand as in the other, how ſhall we know how much is in each hand?*

Bid him put two out of the left into the right-hand, which by that means will have 4 more than the left, and aſk for the Ratio of the number of pieces in the right to that in the left, which we ſhall here ſuppoſe to be as 5 to 3. Then multiply 4, the difference of the two hands, by 3, the leaſt term of the given Ratio, and divide the product 12, by 2, the difference of the two terms of the Ratio 5, 3: the quotient 6 is the number of pieces in the left, to which if you add the difference 4, you have 10 for the right. Theſe two put together make 16, and conſequently at firſt the man had 8 in each hand.

SPORT XXV.

Two perſons having agreed to take at pleaſure leſs numbers than a number propoſed, and to continue it alternately, 'till all the numbers make together a determined number greater than the number propoſed, it is required how to do it?

SUPPOSE the firſt is to make up 100, and both he and the ſecond are at liberty to take alternately any number under 11; let the firſt take 11 from 100 as often as he can, and theſe numbers will remain, 1, 12, 23, 34, 45, 56, 67, 78, 89, which he is to keep in mind; and firſt take 1, for then let the ſecond take what number he will (under 11) he cannot hinder the firſt to come at the ſecond number 12, for if the ſecond takes 3, for example, which, with 1 makes 4, the firſt has nothing to do but to take 8, and ſo reach 12. After that, let the ſecond perſon take what number he will he cannot hinder the firſt from coming at the third number 23; for if he takes 1, for inſtance, which with 12 is 13, the firſt takes 10, and ſo makes 23. In like manner, the firſt cannot be hindered to reach the fourth number 34, then the fifth 45, then 56, then 67, then 78, then 89, and at laſt 100.

As for the ſecond perſon, he can never touch at 100, if the firſt underſtands the way. Indeed if the firſt takes 2 at the beginning, his buſineſs is to take 10, and ſo clap in upon 12, with the ſame advantage the firſt had above. But if the firſt is acquainted with the artifice, he will be ſure to take 1, and ſo the ſecond can never make 12, nor 23, *&c.* nor, in fine 100

If the firſt would be ſure to win, he muſt take care that the leſſer number propoſed does not meaſure the greater; for if it does, he has no infallible rule to go by. For example, if, inſtead of 11, 10 were the number propoſed; taking 10 continually from 100, you have theſe numbers, 10, 20, 30, 40, 50, 60, 70, 80, 90; now the firſt being obliged to pitch under 10, cannot hinder the other from making 10, and ſo 20, 30, *&c.* and in fine 100.

You need not be at the pains to make a continued ſubtraction of the leſſer number from the greater, in order to know the numbers the firſt is to run upon; for if you divide the greater by the leſſer, the remainder of the diviſion is the firſt number you are to take. Thus divide 100 by 11, 1 is the remainder for the firſt number, add to that 11, it makes 12 for the ſecond, and 12 with 11 makes 23 for the third, and ſo on to 100.

SPORT XXVI.

To divide a given number into two parts, the Ratio of which is equal to that of two numbers given.

SUPPOSE 60 is to be divided into two numbers, the leaſt of which muſt be to the greater as 1 to 2: add together the two terms of the given Ratio 1, 2, and divide 60 by their ſum 3: the quotient 20 is the leaſt number wanted, and that ſubtracted from 60 leaves 40 the greater. Or, multiply the two terms 1, 2, ſeparately, by 60, and divide each of the products, 60, 120, by 3, the ſum of the terms; and the two quotients, 20, 40, are the numbers you look for. This Sport gives an eaſy ſolution to the following queſtion:

Queſtion. *To divide the value of a crown into two different ſpecies or denominations, the number of which ſhall be equal.*

The ſolution being demanded in integers, it is impoſſible to ſolve this or the like Sport, unleſs the ſum of the two terms of the Ratio of the different ſpecies propoſed does exactly divide the crown when reduced to ſmaller money. Thus it is impoſſible to divide an Engliſh crown according to the tenour of the queſtion, into ſhillings and pence; becauſe the Ratio of theſe ſpecies or denominations is 12, 1: and 13, the ſum of theſe two terms, does not exactly divide 60 pence, the value of the crown: but make the two ſpecies pence and farthings it will do; ſince 4, 1, the terms of their Ratio, make together 5, which exactly divides 240, the value of the crown in farthings; and the quotient 48, ſolves the queſtion, that is, 48 pence, and 48 farthings make a crown.

SPORT XXVII.

To find a number which being divided by given numbers ſeparately, leaves 1 the remainder of each diviſion; and when divided by another number given, leaves no remainder.

TO find a number which leaves 1 remaining, when divided by 5 and by 7, and nothing when divided by 3. multiply into one another the two firſt numbers given, 5, 7; to their product 35, add 1, which makes 36, the number demanded. For if you divide 36 by 5 and by 7, the remainder is 1; and when you divide it by 3, there is as it happens, no remainder.

After finding this firſt and loweſt number of the propoſed quality 36, you may find an infinite quantity of greater numbers of the ſame quality, and that in the following manner: Add the firſt number found 36, to 105, the product of the three given numbers 5, 7, 3, and the ſum 141 is a ſecond number of the ſame quality, then add to 141 the product above-mentioned 105, and you have 246 for a third; which, with the addition of 105, makes 351 for a fourth number; and ſo on.

To find a number that divided ſeparately by 2, 3, 5, leaves 1 remaining, and no remainder when divided by 11: if you take 30, the product of the firſt three numbers 2, 3, 5, and add 1 to it, you have the number 31, which divided by each of the three firſt numbers, 2, 3, 5, there ſhould remain 11, and by 11, the fourth number, nothing: but ſo it is, that 31, when divided by 11, leaves 9 remaining, and therefore 31 is not the right number; but in order to find out the right number, take 30 the product of the three terms, 2, 3, 5, and quadruple it, which makes 120, which with the addition of 1, is the number required 121, and that added to 1320, the product of the four numbers given 2, 3, 5, 11, makes 1441 for a ſecond number of the ſame quality; and ſo on, as above. In this caſe 30, the product of 2, 3, 5, being divided by 11, left 8 remaining, and the quadruple of that 8, 32, being but 1 ſhort of 33, the multiple or tripple of 11, we quadrupled the 30, and added to the ſum.

In like manner, to find a number that divided ſeparately by 3, 5, 7, leaves 2 remaining, and no remainder when divided by 8: divide 105, the product of the three firſt numbers 3, 5, 7, by the fourth 8; and becauſe there remains 1, multiply the product 105 by 6, that the product 630 divided by 8 may leave a remainder of 6, which is leſs than 8 by 2, and then adding 2 to the laſt product 630, you have 632 the number required, which added to the product of the four given numbers, makes a ſecond number of the ſame quality; and that with the ſame addition, a third, and ſo on.

To find a number that divided ſeparately by 3, 5, 7, leaves 2 remaining, and divided by 11, leaves no remainder: divide 105, the product of the firſt three numbers given, 3, 5, 7, by the fourth 11; and in regard there remains 6, the double of which, 12, ſurpaſſes the diviſor 11 by 1; multiply the product 105 by 2, that 210 being divided by 11, there may remain 1; and ſince it is deſired that 9 may be the remainder, which is leſs than the diviſor 11 by 2, multiply the laſt product 210 by 9, and then the product 1890 being divided by 11, the remainder will be 9; and therefore adding 2 to that laſt product, you will have a number 1890, which leaves no remainder, being divided by 11.

In like manner to find a number that being divided by 5, or 7, or 8, leaves 3 remaining, and nothing when divided by 11: multiply by 9, 280 the product of the firſt three numbers given, 5, 7, 8, and the product 2520 being divided by 11, there remains 1, upon which you may make the remainder 8, which is leſs than 11 by the given number 3, by multiplying the foregoing product 2520 by 8, which makes 20163, and conſequently that ſum, with the addition of 3, *viz.* 20163, is the number ſought for. This Sport directs us to ſolve the following Queſtion:

Queſtion. *To find how many piſtoles were in a purſe that a man has loſt, but remembers, that when he told them by two's, by threes, or by fives, there always remained an odd one, and when he counted them by ſevens, there remained none.*

Here we are to find a number that, when divided by either 2, or 3, or 5, ſtill leaves 1 remainder; and when divided by 7 leaves 0. Now there are ſeveral numbers of that quality, as appears from the foregoing Sport; and therefore to find the number that really was in the purſe, it behoves us to be directed by the bulk or weight of the purſe, in order to determine the real number.

Now to find the leaſt of all theſe numbers, let us firſt of all try for a number that is exactly diviſible by 2, by 3, and by 5, and likewiſe by 7 when 1 is added to it. If you multiply together the three firſt numbers given, 2, 3, 5, their product 30 will be diviſible by each of theſe three numbers; but when you have added 1 to it, the ſum 31 is not diviſible by the fourth number given, 7, for there remains 3, and ſince the product 30, when divided by 7, leaves 2, its double 60 will leave 4 upon the like diviſion, and by the ſame conſequence its tripple 90 will leave 6 remaining. Now 6 wanting but one of 7, add that one to this tripple number 90, and ſo 91 will be exactly diviſible by 7, and conſequently is the number ſought for.

To find the next larger number that anſwers the Queſtion, multiply together the four given numbers 2, 3, 5, 7, and to their product 210 add the firſt and leaſt number found 91; the ſum 301 is the ſecond number ſought for; and if you add to this ſecond number the foregoing product 210, the ſum 511 will be the third number that ſolves the Queſtion; and ſo on *in infinitum.*

Thus, to reſolve the Queſtion, you may anſwer, that there might be in the purſe 91 Louis d'Ors, or 301, or 511; and the bulk of the purſe will ſerve to direct you which of the numbers was really in it.

SPORT XXVIII.

Of ſeveral numbers given to divide each into two parts, and to find two numbers of ſuch a quality, that when the firſt part of each of the given numbers is multiplied by the firſt number given, and the ſecond by the ſecond, the ſum of the two products is ſtill the ſame.

SUPPOSE, for example, theſe three numbers given, 10, 25, 30, and the ſolution is required in entire numbers; take any two numbers for the two numbers ſought for, provided their difference be 1, or ſuch as may exactly divide the product under the greateſt of theſe two numbers, and the difference of any two of the three given numbers, and ſo, that the greateſt of theſe two numbers multiplied by the leaſt given number 10, may be greater than the leaſt of theſe two numbers multiplied by the greateſt given number 30; ſuch are 2 and 7.

The two numbers required, 2 and 7, being thus found; the firſt part of the firſt given number 10, may be taken at pleaſure, provided it is leſs than 10, and then the number ariſing from the ſubtraction of the leaſt found number 2, multiplied by the greateſt given number 30, from the greateſt found number 7, multiplied by the leaſt given number 10; and then the number ariſing from the diviſion of the remainder 10 by 5, the difference of the two numbers found 2, 7; that is, leſs than 2, which is 1, which being ſubtracted from the firſt given number 10, leaves the remainder 9 for the other part; and that being multiplied by the ſecond number found 7, and the firſt part 1 being multiplied by the firſt number found 2, the ſum of the two products 63 and 2 is 65.

To find the firſt part of the ſecond number given, 25, multiply 15, the difference of the firſt two numbers given, 10, 25, by the greateſt number found 7; and divide the product 105 by 5, the difference of the two numbers found 2, 7; then add the quotient 21 to 1, the firſt part found of the firſt number given 10; and the ſum 22 will be the firſt part of the ſecond number given 25, and conſequently the other part will be 3, which being multiplied by the ſecond number found 7, and the firſt part 22, being multiplied by the firſt number given, 2, the ſum of their two products 21, 44, makes likewiſe 65.

Laſt of all, to find the firſt part of the third number given 30, multiply 5, the difference of the two laſt numbers given, 25, 30, by the greateſt number found, 7, and divide the product 35 by 5, the difference of the two numbers found 2, 7; then add the quotient 7 to 22, the firſt part of the ſecond number given 30, 30, and the ſum 29 will be the firſt part of the third number given 30, and conſequently the other part will be 1, which being multiplied by the ſecond number found 7, and the firſt part 29 being multiplied by the firſt number found 2, the ſum of the two products 7, 58, makes ſtill 65.

Or elſe multiply 20, the difference of the firſt and the third number given, by the greateſt number found 7, and divide the product 140 by 5, the difference of the two numbers found 2, 7; then add the quotient 28 to 1, the firſt part of the firſt number given 10, and you will have 29 as above, for the firſt part of the third number given 30.

If you take 1 and 6 for the two numbers ſought for, and 4 for the firſt part of the firſt number given 10, in which caſe the other part will be 6, which being multiplied by the ſecond number found, 6, and the firſt part 4 by the firſt number found 1, the ſum of the two products 36 and 4, is 40. Upon this ſuppoſition, I ſay, the firſt part of the ſecond number given 25, will be 22, and conſequently the other part 3, which being multiplied by the ſecond number found 6, and the firſt part 22 by the firſt found number 1, the ſum of the two products 18, 22, is likewiſe 40; and in fine, the firſt part of the third number given 30, will be 28, and the other 2, which being multiplied by the ſecond number given 6, and the firſt 28 by the firſt 1, the ſum of the two products is ſtill 40.

This Sport directs us to the ſolution of the following Queſtion:

Queſtion. *One woman ſold at market* 10 *apples at a certain rate apiece; another ſold* 25 *at the ſame rate; and a third ſold* 30 *ſtill at the ſame price; and yet each of them brought the ſame ſum of money home with them. The queſtion is how this could be?*

It is manifeſt, that to ſave the poſſibility of the Queſtion, the women muſt ſell their apples at two different ſales, and at two different rates, ſeeing at each ſale or diviſion they ſell at the ſame rate Let the two different rates be 2 and 7, which are the two numbers that we found in the foregoing Sport; and we will ſuppoſe

	Apples		*Farthings*	*Apples*		*Farth.*	
X.	1	at	2	9	at	7	65
XXV.	22	at	2	3	at	7	
XXX.	29	at	2	1	at	7	

that at the firſt ſale they ſold at 2 farthings an apple, and that at this rate the firſt ſells 1 apple, the ſecond 22, and the third 29; the three numbers 1, 22, 29, being the firſt parts of the three given numbers X, XXV, XXX, which were found in the foregoing Sport; in this caſe the firſt woman will take 2 farthings, the ſecond 44, and the third 58. In the next place, if we ſuppoſe they ſell the reſt of their apples at 7 farthings, then the firſt woman will take 63 farthings for the 9 apples ſhe had left, the ſecond will take 21 farthings for the 3 apples ſhe had left, and the third 7 farthings for the 1 apple ſhe had left; and ſo each of them will take in all 65 farthings.

Or, if you will, make the two different rates 1 and 6, which were the two numbers found in the laſt Sport; and ſuppoſe at the firſt ſale they ſell at a farthing an apple, at which price the firſt ſells 4, the ſecond 22,

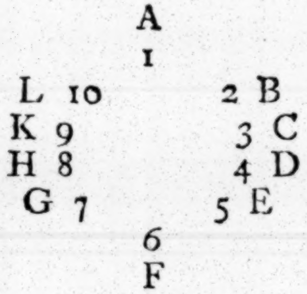
	Apples		*Farthings*	*Apples*		*Farth.*	
X.	4	at	1	6	at	6	40
XXV.	22	at	1	3	at	6	
XXX.	28	at	1	2	at	6	

and the third 28; theſe three numbers 4, 22, 28, being the firſt parts of the given numbers X, XXV, XXX, which were found in the laſt Sport; the firſt woman will take 4 farthings, the ſecond 22, and the third 28. Then ſuppoſe again that they ſell the reſt of their apples at 6 farthings a piece, the firſt woman will take 36 farthings for the 6 apples ſhe had left, the ſecond 18 for the 3 apples ſhe had left, and the third 12 farthings for the 2 apples ſhe had left. And thus every one of them will take in all 40 farthings.

SPORT XXIX.

Many numbers which proceed from 1 *or unity, in a progreſſion, according to the natural order of numbers, (ſuch as theſe,* 1, 2, 3, 4, 5, 6, *&c.) being placed in a round form like a ring; to diſcover which of theſe numbers any one has thought upon.*

LET any multitude of numbers in the aforeſaid progreſſion, ſuppoſe theſe ten, to wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, be marked upon ten ivory-counters (or for want thereof upon ten ſmall pieces of paper) which may be repreſented by theſe ten letters, A, B, C, D, E, F, G, H, K, L, *viz.* ſuppoſe 1 to be writ upon the counter A, 2 upon B, 3 upon C, *&c.* Then having placed thoſe counters circularly as you ſee (with their blank faces uppermoſt, and the figures underneath, that the ſubtility of the Sport may the better be concealed) let any one think upon any number of units which does not exceed ten; that done, bid him touch one of thoſe counters at pleaſure, and to the number on the back ſide of the counter touched (which you cannot be ignorant of, having noted well the place of 1 or A) add ſecretly in your mind, the juſt number of all the counters and reſerve the ſum; then bid him imagine in his mind the counter touched to be the number which he thought, and from that counter to count backwards, until he has made up the aforeſaid ſum, which you reſerved, ſo will his com-



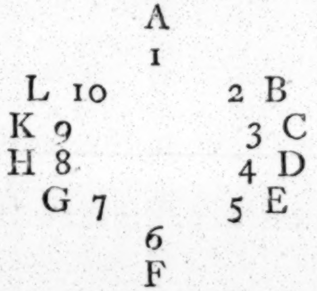
computation infallibly end on the counter upon which the number thought of is marked.

For example, ſuppoſe he thought 7 or G, and that he touched B, to wit, 2; add to 2 the number of all the counters, to wit, 10, ſo the ſum will be 12; then bid him count to 12, beginning at B and going backward, and eſteeming B to be the number thought, to wit, 7; ſo will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H; and laſtly, 12 upon the counter G, which being turned up will ſhew the number thought.

The reaſon of this rule is not difficult to be apprehended, two principles being pre-ſuppoſed the one is this; to wit, many counters or things whatſoever being diſpoſed orderly one after the other, in one continued line, whether it be right or circular; if you value or name the firſt counter to be ſome number of units at pleaſure, and continue the count forward according to the natural order of numbers, until another number be named which falls upon the laſt counter: Or, if you imagine or name the laſt counter, to be the ſame number of units as before you put upon the firſt, and continue to count backwards to the firſt counter; I ſay that the ſame number will be named at the end of both theſe computations: For example, in theſe 9 letters, A, B, C, D, E, F, G, H, K, if the letter A be eſteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers, the letter K will fall upon the number 12. In like manner, if you eſteem K to be 4, and count backwards from K to A, the letter A will likewiſe fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12.
A.	B.	C.	D.	E.	F.	G.	H.	K.
12.	11.	10.	9.	8.	7.	6.	5.	4.

The other principle is this, to wit, many counters being diſpoſed in a round form like a ring; if you eſteem any one of thoſe counters to be ſome number at pleaſure, and then from that counter if you count circularly, until you end upon the counter where you began, the number laſt named will be equal to the ſum of the number of all the counters, and of the number which you put upon the firſt counter; for example, if D be one of the ten letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural progreſſion of numbers till you end with



D where you began; the number 17, which is compoſed of 10 and 7, will neceſſarily fall upon D; for 9 (which is the number of letters in the circumference beſides D) being added to 7 (which was firſt put upon D) makes 16, to which 1 being added, (becauſe D ends as well as begins the circumference) the ſum is 17.

Now theſe two principles being pre-ſuppoſed, it will not be difficult to apprehend the reaſon of the aforeſaid Rule in all caſes that can happen; for imagine that one has thought upon 7, or the counter G, then that counter which he ſhall touch muſt either be the ſame counter G, or ſome other that precedes or follows G.

Firſt therefore, ſuppoſing the counter or number touched to be the ſame with the number thought, the truth of the Rule will be then evident; for by the Rule given, he will begin to count from the ſame G to 17, putting 7 upon G, therefore by the ſecond preſuppoſition the number 17 will fall upon G.

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10; then according to the Rule adding 10 (the numbers of all the counters placed circularly) to 10 or L, the (counter touched (bid him count backwards to 20 by beginning at L, and eſteem L to be 7. Now, becauſe by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L; therefore by the firſt preſuppoſed principle, if we eſteem L to be 7, and count backwards, the number 10 will infallibly fall upon G, and then the number 20 ſhall alſo fall upon the ſame G by the ſecond preſuppoſed principle.

Laſtly, imagine he touched ſome number or counter which precedes 7 the number ſought, as B or 2; then adding 10 to 2, you are to bid him count unto 12, he having firſt imagined B to be the number thought 7; and going backward to A, L, K, *&c.* Now becauſe by proceeding to count at B, which is 2, and beginning to count forward to C, D, *&c.* the number 7 falls upon G; therefore if one imagine that G is 2, and from thence count backwards towards E, F, *&c.* the number 7 will fall upon B (by the firſt preſuppoſed principle) therefore when one aſſumes B to be 7, and counts towards A, L, *&c.* to any aſſigned number, it is in effect as much as when one imagines G to be 2, and counts towards E, F, unto the ſaid aſſigned number, for each of thoſe computations will end in the ſame point; but it is manifeſt (by the ſecond preſuppoſed principle) that eſteeming G to be 2, and counting towards E, F, D, *&c.* round the whole circumference; the number 12 will fall upon the ſame G. And becauſe G being ſuppoſed to be 2, and counting on the ſame coaſt as before, the number 7 falls upon B; therefore if the computation be continued on the ſame coaſt as before, the number 7 falls upon B; therefore if the computation be continued on the ſame coaſt from B 7, to 12, the number 12 will fall upon the ſame G. So that the practice of this Sport in all its caſes is demonſtrated.

Note, That to the number of the counter touched, you may not only add the number of all the counters once (as the Rule directs) but twice, thrice or more times: For example, B being touched, you may cauſe him to count to 12, or 22; or to 32, 42, *&c.* the reaſon whereof is evident from the ſecond preſuppoſed principle.

SPORT XXX.

Many numbers being ſhewed by pairs, to wit, two by two unto any perſon, that he may think upon any one of theſe pairs at pleaſure, to diſcover the pair that was thought upon.

LET twenty numbers, ſuppoſe theſe, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, be writ upon ivory-counters (or for want thereof upon ſmall pieces of paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, *&c.* Then diſpoſe them into pairs, as you ſee, *viz.* ſuppoſe 1 and 2 to be one pair, 3 and 4 to be another pair, *&c.* and of theſe pairs let any one think upon which pair he pleaſes. That done, you are to diſtribute the ſaid 20 numbers into ranks, in form of a long ſquare, until there be 5 numbers in length, and 4 in breadth, after this manner, *viz.* lay the three firſt numbers, 1, 2, and 3, in a rank (as you ſee in the ſecond figure) from A towards B; then place 4 under 1, and 5 after 3, (in the ſaid rank A, B) Again, place 6 under 4, and 7 after 5, (in the ſaid rank A, B). Then place 8 under 6, alſo 9, 10, 11, on

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20

on the right-hand of 4, in the rank C, D. Again, place 12 under 9, and 13 on the right-hand of 11, in the rank C, D, and 14 under 12. Moreover, placing 15, 16, 17, on the right-hand of 12, in the rank E, F. Laſtly, place 18, 19, 20, on the right-hand of 14, in the rank G, H, ſo will all the numbers be ranked as you ſee in the Table. That done, you are to demand of him that thought upon two numbers as aforeſaid, in what rank or ranks the ſaid numbers happen to be found, *viz.* in which of the ranks A B, C D, E F, G H, or in which two of the ſaid ranks: Now if he anſwer

A	1	2	3	5	7	B
C	4	9	0	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

that the two numbers he firſt thought upon are in the firſt rank A, B, then 1 and 2 will be the numbers thought or kept in mind; if in the ſecond C D, then 9 and 10 ſhall be the numbers thought of; if in the third rank E F, then 15 and 16 will be the numbers thought: If they are in the fourth rank G H, then 19 and 20 ſhall be the numbers thought; but if he ſay that the numbers thought are in different ranks, then you are heedfully to mark the ſaid numbers 1, 2, 9 and 10, 15 and 16, 19 and 20, which may be called the keys of the ſport, in regard they ſerve not only to diſcover the two numbers thought, when they are both in one and the ſame rank (as aforeſaid) but even when they are in two different ranks: For in this latter caſe, as ſoon as it hath been declared to you in which two ranks the two numbers thought are placed, you are to take the key of the higheſt of thoſe two ranks, and deſcending in a down right line from the firſt number of that key to the lower of the ſaid two ranks, you'll there find one of the two numbers thought, and upon the right-hand of the ſecond number of the ſaid key, at the ſame diſtance ſideways, from the ſecond number of the key, (as one of the numbers thought was diſtant from the firſt number of the key) you will find the other number thought.

For example, ſuppoſe the two numbers thought are 7 and 8, and it is declared to you, that they are in the firſt and fourth ranks; take then the key of the higheſt of thoſe two ranks, to wit, of the firſt, which is 1 and 2, and deſcending downright from 1 to the fourth rank, you'll there find 8 one of the numbers thought: then ſeek ſideways on the right-hand of 2 (the ſecond number of the key) a number as far ſeparated from 2, as 8 is diſtant from 1, and you'll find 7 the other number thought.

Again, ſuppoſe he ſays that the numbers thought are in the ſecond and third ranks: Take then the key of the ſecond rank which is 9 and 10, and deſcending downright from 9 to the third rank, you ſhall there find 12, which is one of the numbers thought; then ſeek ſideways on the right-hand of 10, (the ſecond number of the key a number as far diſtant from 10, as 12 is from 9, and you'll find 11, which is the other number thought.

The reaſon of this will be apparent, from a ſerious conſideration of the placing of the numbers according to the Rules before given: for it is thereby evident, that of the firſt numbers coupled two by two, there can never be found more than one pair in one and the ſame rank; and of all the other pairs, one number is always found in one rank, and the other number in another rank.

Note alſo, that this Sport may be practiſed with divers perſons at once, and not only with 20 numbers, but with any ſuch multitude of numbers as is produced by the multiplication of any two numbers that differ by 1, or unity; as 30, which is the product of 5 multiplied by 6, and 42, which is the product of the multiplication of 6 and 7. That which is chiefly to be regarded is, the placing of the numbers in ranks according to the directions before given: And for the more eaſy comprehending of that order, I have, in the following Table, ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer illuſtration than can be expreſſed by many words.

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

SPORT XXXI.

Among three perſons, to find how many cards or counters each of them has got.

LET the third perſon take what number of cards or counters he pleaſes, provided it be evenly even, that is, diviſible by 4; let the ſecond take as many 7's as the other has taken 4's; and the firſt as many 13's. Then bid the firſt give to the other two as many of his counters as each of them had before; and the ſecond to give to the remaining two as many of his counters as each of them; and in like manner, the third to give to each of the other the ſame number that they have. By this means it will ſo fall out, that they will all have the ſame number of counters, and each of them will have double the number that the third had at firſt. And for this reaſon, if you aſk one of the three how many counters he has got, half his number is the number the third had at firſt; aad if you take as many 7's, and as many 14's as there were 4's in the third perſon's number, you will have the number of cards or counters that the ſecond and firſt took.

For example, if the third took 8 cards, it behoved the ſecond to take 14, that is twice 7, becauſe there is twice 4 in 8; and the firſt muſt take 26, that is twice 13 by the ſame reaſon. If the firſt, who has 26 cards, gives to the ſecond 14, that is, as many as he had at firſt: and to the third 8, that being his firſt number,

1ſt	2d.	3d.
26	14	8
4	28	16
8	8	32
16	16	16

number, he will have only 4 left to himſelf; and the ſecond will have 28; and the third 16. But if the ſecond, who had 28 cards, gives out of his cards 4 to the firſt, who had juſt as many before, and 16 to the third, who had likewiſe as many; he will have 8 left to himſelf, and the firſt will have 8, and the third 32. In fine, if the third, who has got 32, gives 8 to each of the others, all the three will have 16, which is the double of 8, the number that the third took up at firſt.

SPORT XXXII.

Of three unknown cards, to find what card each of three perſons has taken up.

THE number of each card taken up muſt not exceed 9. Then, to find out that number, bid the firſt ſubtract 1 from double the number of the points of his card, and after multiplying the remainder by 5, add to the product the number of the points of the ſecond perſons card. Then cauſe him to add to that ſum 5, in order to have a ſecond ſum; and after he has taken 1 from the double of that ſecond ſum, make him to multiply the remainder by 5, and add to the product the number of the points of the third perſon's card. Then aſk him the ſum ariſing from this laſt addition; for if you add 5 to it, you will have another ſum compoſed of three figures, the firſt of which towards the left is the number of the points of the card that the firſt perſon took up; the middle figure will be that of the ſecond perſon's card; and the laſt towards the right, directs you to the third perſon's card.

For example; if the firſt took a 3, the ſecond a 4, and the third a 7; by taking 1 from 6, the double of the firſt 3, and multiplying the remainder 5 by 5, we have 25 the product, to which we add 4, the number of the ſecond perſon's card, which makes 29, and that with the addition of 5, makes the ſecond ſum 34, the double of which is 68, and taking 1 from that, there remains 67, which being multiplied by 5, makes 335, and this by the addition of 7, the number of the third perſon's card, and 5 over and above, makes the laſt ſum 347, the three figures of which ſeverally repreſent the number of each card.

Another way, Or, if you will, you may bid the firſt add 1 to the double of the number of the points of his card, and multiply the ſum by 5, and add to the product the number of the ſecond perſon's card. Then bid him add in like manner 1 to the double of the preceding ſum, and multiply the whole by 5, and add to the product the number of the third perſon's card. Then aſk him the ſum ariſing from the laſt addition, and ſubtract 55 from it, that ſo there may remain a number compoſed of three figures, each of which repreſents, as above, the number of each card.

As in the foregoing example, by adding 1 to 6, the double of 3, the number of the firſt perſon's ſon's card, and by multiplying the ſum 7 by 5, we have 35, which, with the addition of 4, the number of the ſecond perſon's card, makes 39, the double of which is 78, to which if we add 1, and multiply the ſum 79 by 5, we have 395; to that we add 7, the number of the third perſon's card, and ſo have 402, from which if we ſubtract 55, the remainder is 347, the three figures of which ſeverally repreſent the number of each card.

SPORT XXXIII.

Of three cards known, to find which and which is taken up by each of three perſons.

OF the three known cards, we ſhall call one A, the other B, and the third C, and leave each of the three perſons to pitch upon 1 of the three, which may be done ſix different ways, as you ſee in the annexed ſcheme. Give the perſon the number 12, the ſecond 24, the third 36— Then direct the firſt perſon to add together the half of the number of that perſon that has taken the card A, the third part of the number of the perſon that takes the card B, and the fourth part of the number of the perſon that takes the card C; and

1ſt.	2d.	3d.	*Sums.*
12	24	36	
A	B	C	23
A	C	B	24
B	A	C	25
C	A	B	27
B	C	A	28
C	B	A	29

and then aſk him the ſum, which you will find to be either 23 or 24, or 25, or 27, or 28, or 29, as you ſee in the table or ſcheme which ſhews, that if the ſum is, for example, 25, the firſt will have taken the card B, the ſecond the card A, and the third the card C; and if the ſum is 28, the firſt has taken the card B, the ſecond the card C, and the third the card A; and ſo on in the other caſes.

SPORT XXXIV.

To find out among ſeveral cards, one that another has thought of.

HAVING taken out of a pack of cards a certain number of cards at pleaſure, and ſhewn them in order upon the table, before the perſon that is to think, beginning with the lowermoſt, and laying them cleverly one above another, with their figures and points upwards, and counting them readily, that you may find out the number; which, for example, we ſhall here ſuppoſe to be 12; bid him keep in mind the number that expreſſes the order of the card he has thought of, namely 1, if he has thought of the firſt, 2, if he has thought of the ſecond, 3, if he has thought of the third, &c. Then lay your cards one after another, upon the reſt of the pack, in a contrary ſituation, and putting that upon the pack firſt that was ſhewn upon the table, and that laſt that was laſt ſhewn. Then aſk the number of the card thought of, which we ſhall here ſuppoſe to be 4, that is, the fourth card in order of laying down, is the card thought of. Lay your cards, with their faces up, upon the table, one after another, beginning with the uppermoſt, which you are to reckon 4, the number of the card thought of; ſo the ſecond next to it will be 5, and the third under that 6, and ſo on till you come to 12, the number of the cards you firſt pitched upon to ſhew the perſon; and you will find the card that the number 12 falls to, to be the card thought of.

SPORT XXXV.

Several parcels of cards being propoſed or ſhewn, to as many different perſons, to the end that each perſon may think upon one, and keep it in his mind; to gueſs the reſpective card that each perſon has thought of.

WE will ſuppoſe there are 3 perſons, and 3 cards ſhewn to the firſt perſon, that he may think upon one of them, and theſe three cards laid aſide by themſelves; then 3 other cards held before the ſecond perſon, for the ſame end, and laid apart; and at laſt, 3 different cards again to the third perſon, for the ſame end, and likewiſe laid apart. This done, turn up the 3 firſt cards, laying them in three ſtations; upon theſe three lay the next three other cards that were ſhewn to the ſecond perſon; and above theſe again the leaſt three cards.

Thus you have your cards in three parcels, each of which conſiſts of 3 cards. Then aſk each perſon in what liſt is the card he thought of; after which it will be eaſy to diſtinguiſh it; for the firſt perſon's card will be the firſt of his heap; and in like manner the ſecond's will be the ſecond in his; and the third perſon's card will be the third in his.

SPORT XXXVI.

Several cards being ſorted into three qual heaps, to gueſs the card that one thinks.

IT is evident that the number of cards muſt be diviſible by 3, ſince the three liſts are equal. Suppoſe then there are 36 cards, by conſequence there are 12 in each liſt: aſk in what liſt is the card thought upon; then put all the heaps together, ſo as to put that which contained the card thought upon between the other two; then deal off the 36 cards again into three equal hands, obſerving that order of the firſt card to be firſt, the ſecond to the ſecond, the third to the third, the fourth to the firſt again, and ſo round, dealing one card at a time, till the cards are dealt off. Then aſk again, in what hand or heap is the card thought upon, and after laying together the cards, ſo as to put that liſt which contained the card between the other two, deal off again, as you did before, into three equal liſts. Thus done, aſk once more, what liſt the card is in, and you will eaſily diſtinguiſh which is it, for it lies in the middle of the liſt to which it belongs; that is, in this example, it is the ſixth card; or, if you will, to cover the artifice the better, you may lay them all together, as before, and the card will be in the middle of the whole, that is, the eighteenth.

SPORT XXXVII.

To gueſs the number of a card drawn out of a complete ſtock.

AFTER one hath drawn what card he pleaſes out of a complete ſtock of 52 cards, for inſtance, ſuch as we play at *Ombre* with, you may know how many points are in the card thus drawn, by reckoning every faced card 10, and the reſt according to the number of their points; then looking upon the reſt of the cards one after another, add the points of the firſt card to the points of the ſecond, and the ſum to the points of the third, and ſo on, till you come to the laſt card, taking care all along to caſt out 10, when the number exceeds it; upon which account you ſee it is needleſs to reckon in the 10's or the faced cards, ſince they are to be caſt out however. Then if you ſubtract your laſt ſum from 10, the remainder is the number of the drops of the card drawn.

It is eaſy to know that when nothing remains the card drawn is either a 10 or a faced card: and that in this caſe, if it be a faced card, one cannot diſtinguiſh whether it be a king, queen or knave: now, in order to be maſter of that diſtinction, the beſt way is to make uſe of a ſtock of 36 cards only, ſuch as we formerly uſed for Piquet, and reckon a knave 2, a queen 3, and a king 4.

If you make uſe of a ſtock of 32 cards only, ſuch as is now uſed for Piquet, you are to follow the ſame courſe as is above preſcribed, only you muſt always add 4 to the laſt ſum, in order to have another ſum, which being ſubtracted from 10 if it be leſs, or from 20, if it ſurpaſſes 10, the remainder will be the number of the card drawn; ſo that if 2 remains it is a knave, if 3 a queen, if 4 a king, &c.

If the ſtock is not full, you muſt take notice what cards are wanting, and add to the laſt ſum the number of all the cards that are wanting, after ſubtracting from that number as many 10's as are to be had; upon which, the ſum ariſing from this addition is to be ſubtracted, as above, from 10 or from 20, according as it is above or under 10. This done, it is evident by caſting your eye once more upon the cards, you may tell what card was drawn.

SPORT XXXVIII.

To gueſs the number of the points or drops of two cards drawn out of a complete ſtock of cards.

LET a man draw at pleaſure two cards out of a ſtock of 52 cards; bid him add to each of the cards drawn as many other cards as his number is under 25, which is the half of all the cards, wanting 1, fixing upon each faced card what number he pleaſes; as if the firſt card be 10, add to it 15 cards; and if the ſecond card be 7, add to it 18 cards; ſo that in this example there will remain but 17 cards in the ſtock, the whole number taken out amounting to 35. Then taking the remainder of the pack into your hands, and finding they are but 17, conclude that 17 is the joint number of all the points of the two cards drawn.

To cover the artifice the better, you need not touch the cards, but order the drawer to ſubtract the number of the points of each of the two drawn cards from 26, which is half the number of all the cards, and direct him to add together the two remainders, and acquaint you with the ſum, to the end you may ſubtract it from the number of the whole ſtock, *i. e.* 52; for the remainder of that ſubtraction is what you look for.

For *Example.* Suppoſe a 10 and a 7 are the cards drawn; take 10 from 26, there remains 16; and taking 7 from 26, the remainder is 19: the addition of the two remainders 16, 19, makes a ſum of 35, which ſubtracted from 52, leaves 17 for the number of the drops of the two cards drawn.

The ſame is the management in a ſtock of 36 or 32 cards; only to colour the trick the better, inſtead of 26, the half of the cards, when they make 52, take another leſſer number, but greater than 10, as 24, from which taking 10 and 7, there remains 14 and 17, the ſum of which, 31, being ſubtracted from 52, the ſum of all the cards, leaves 21 the remainder; from which ſubtract again 4, which is the double of the exceſs of 26 above 24, and ſo the remainder is 17, the number of the points of the two cards drawn, *viz.* 10 and 7.

If you make uſe of a Piquet-ſtock, conſiſting of 36 cards, inſtead of 18, the half of 36, the number of all the cards, take in like manner a leſſer number, ſuch as 16, from which take 10 and 7, and there remains 6 and 9, the ſum of which, 15, being ſubtracted from 36, the number of all the cards, leaves 21 remaining; from which ſubtract again 4, the double of the exceſs of 18 above 16, and ſo the 17 remaining is the number of the points of the two cards drawn.

In like manner, if this Piquet ſtock conſiſts only of 32 cards, inſtead of 16, the half of 32, the number of the whole ſtock, take any leſſer number you will, provided it be greater than 10, ſuch as 14, from which take 10 and 7, and the remainders are 4 and 7, the ſum of which, 11, being taken from 32, leaves 21, and taking from that 4, the double of the exceſs of 16 above 14, you have 17 remaining, the number of the points of the 10 and the 7 drawn.

SPORT XXXIX.

To gueſs the number of all the drops of three cards drawn at pleaſure, out of a complete ſtock of cards.

TO ſolve this Sport as the former, after the ſhorteſt way, the number of cards contained in the ſtock muſt be diviſible by 3; ſo that neither a ſtock of 52, nor one of 32, are proper; but one of 36 is, in regard 36, the number of all the cards, has 12 for its third part, which will aſſiſt us in the ſolution of the queſtion, as follows:—

Let a man draw at pleaſure three cards out of a Piquet ſtock of 36 cards; bid him add to each of theſe cards as many other cards as the number of their points falls ſhort of 11, which is the third part of the number of all the cards, wanting one, allotting, as in the foregoing Sport, to each ſacred card what number he pleaſes: as if the firſt card is 9, he adds to it 2 cards; if the ſecond is 7, he adds to it 4; and if the third is 6, he adds 5; which make in all 14 cards; ſo that in this example the remainder of the whole ſtock is 22 cards, which denotes the number of all the points of the three cards drawn.

The better to colour the artifice, you need not touch the card, but bid him ſubtract the number of the points of each of the three drawn cards from 12, the third part of 36, the number of the whole ſtock, and add together the three remainders, and tell you the additional ſum, which you are to ſubtract from 36, and the remainder of that ſubtraction is what you look for.

As in this Example—Suppoſe he drew a 9, a 7 and a 6, take 9 from 12, there remains 3; take 7 from 12, there remains 5; and take 6 from 12, there remains 6; add the three remainders, 3, 5, 6, the ſum is 14, which taken from 36, leaves 22 for the number of the drops of the three cards drawn.

To colour the trick the better, and to apply the rule to a ſtock that conſiſts of fewer or more than 36 cards, ſuch as one of 52 cards, make uſe of a number greater than 10, and leſſer than 17, the third part of 52; for inſtance 15: bid him who drew the three cards add to each of his drawn cards as many other cards as the number of their reſpective points is under 15: for example, if the firſt card be 9, he adds to it 6 cards; if the ſecond is 7, he adds 8; if the third is 6, he adds 9; which makes in all 26 cards; ſo that in this example there will remain in the main ſtock 26 cards. Taking the main ſtock into your hands, and finding you have 26 cards, ſubtract from 26 the number 4, which is the exceſs of 52, the number of the whole ſtock, above the tripple of 15+3, *i. e.* 48; and the remainder 22 is the number of all the points of the three cards drawn.

Or elſe you need not touch the cards, but bid the perſon that draws ſubtract the number of the drops of each of the three cards drawn from

16, which is 1 more than the firſt number 15, and add together all the remainders, and acquaint you with the ſum; then do you ſubtract that ſum from the number above-mentioned, 48, and you will find the remainder to be the number of all the points of the three cards drawn.

For example—Suppoſe he drew a 9, a 7, and a 6; take 9 from 16, there remains 7; take 7 from 16, there remains 9; take 6 from 16, there remains 10; add theſe three remainders, 7, 9, 10, the ſum is 26, which ſubtracted from 48, leaves 22 for the number of the points of the three cards drawn.

In like manner, in a pack of 36 cards, take a larger number than 10, for inſtance 15; and taking notice of the additional cards, which amount to 26, as you ſeen but now, ſubtract that number, 26 from 36, the number of the whole pack, and to the remainder 10 add 12, which is the exceſs of the tripple of 15+3, *i. e.* 48, above 36, the number of the whole; and you will find the ſum 22, to be the number of points enquired after In a Piquet-pack or 32 cards, inſtead of 12 you muſt add 16, by reaſon that 16 is the remainder of 32 ſubtracted from 48.

In imitation of this and the foregoing Sport it will be eaſy to ſolve the queſtion upon four or more cards drawn.

SPORT XL.

The Game of the Ring.

THIS is an agreeable game in a company of ſeveral perſons, not exceeding 9 (unleſs you have a mind to it), in order to the eaſier application of the 22d Sport, *viz.* by reckoning the firſt perſon 1, the ſecond 2, the third 3, and ſo on; and in like manner, reckoning the right-hand 1, the left-hand 2; the thumb of the hand 1, the fore-finger 2, the third finger 3, the fourth 4, and the little one 5; the firſt joint 1, the ſecond 2, and the third 3. For if you put the ring to one in the company, for inſtance, the fifth perſon, and that upon the firſt joint of the fourth finger of the left hand, it is evident, that in order to gueſs who has the ring, and upon which hand, which finger, and which joint, one has only theſe four numbers to gueſs, 5, 1, 4, 2, the firſt number 5 repreſenting the fifth perſon; the ſecond 1, the firſt joint; the third 4, the fourth finger; and the laſt 2, the left-hand. Now this is performed by obſerving the laſt method of Sport 22 foregoing, as appears from the following operation.

Taking 1 from 10, the double of the firſt number 5, and multiplying 9, the remainder by 5, you have 45; adding to that the ſecond number 1, you have 46, to which if you add 5, you have 51 for a ſecond ſum: the double of this ſecond ſum is 102, from which take 1, there remains 101, which being multiplied by 5, 5, makes 505, and that with the addition of 4, the third number, makes 509, to which if you add 5, you have this ſecond ſum 514, the double of this 1028 leſſened by 1, and the remainder multiplied by 5 makes 5135, to which adding the fourth number 2, you have this ſum 5137, and that augmented by 5 gives this ſecond ſum 5142, the four figures of which repreſent the four numbers inquired for, and by conſequence denote that the ring is upon the firſt joint of the fourth finger of the left-hand of the fifth perſon.

PART II.

Artificial FIRE-WORKS.

SPORT XLI.

To make Silent Powder, or ſuch as may be diſcharged without a noiſe.

THIS *unſounding powder*, if any ſuch there is, goes commonly under the name of *White-powder*, becauſe, poſſibly, the firſt made was of that colour. It is not probable it can be of any great force, for as much as the noiſe of gun-powder proceeds from the violent percuſſion of the air, occaſioned by the ſtrength of it. I have not indeed ſeen this powder myſelf, yet I have read in authors ſeveral ways of making the ſame, of which the following two only occur to my memory.

The firſt is thus: To one pound of common Gun powder take half as much Venetian Borax, which having pulveriſed, mixed, and well incorporated together, reduce the mixture into grains, and you have the powder required.

The other way is—To four pounds of common Gun-powder, add two pounds of Venetian Borax, one pound of Lapis Calaminaris, and one pound of Sal armoniac; pulveriſe them all together to make of them a powder in grains, as before.

SPORT XLII.

To prepare an Oil of Sulphur required in Fire-works.

HAVING melted what quantity of Sulphur you think fit, upon a moderate fire in an earthen or copper veſſel, throw into it ſome old, or, in defect of this, ſome new brick that is well burned, and was never wetted, broken into many ſmall pieces about the bigneſs of a bean; ſtir them continually with a ſtick, till they have drunk up and conſumed all the ſulphur; this done, ſet them upon a furnace to diſtil in an Alembic; ſo you ſhall have a very inflammable oil, fit for your purpoſe.

You may make it otherways thus—Fill one third or fourth part of a glaſs bottle with a long neck with Sulphur pulveriſed; then pouring upon it ſpirit of Turpentine, or oil of walnuts, or of Juniper, till the bottle is half full, ſet it upon hot cinders, leaving it there eight or nine hours, and you ſhall find an oil therein of the aboveſaid quality.

SPORT XLIII.

To prepare the Oil of Saltpetre uſeful in Fire-works.

PUT upon a fire-board, well plained and dry, what quantity of purified Saltpetre you pleaſe, and cauſe it to melt by putting thereupon burning coals; and you ſhall ſee the liquor to paſs through the board, and to fall down drop by drop, which muſt be received in an earthen or copper pot, where you have an oil of Saltpetre, fit to be uſed in fire-works, as we ſhall declare in its proper place.

SPORT XLIV.

To prepare the Oil of Sulphur and Saltpetre mixed together.

HAVING mixed and well incorporated equal portions of Sulphur and Saltpetre, reduce all into a fine powder, which muſt be paſſed through a fine ſearce; put this powder thus ſearced into a new earthen pot, or one that hath not been uſed, and pour upon it good white-wine vinegar, or elſe brandy, till it is covered. Then cover your pot ſo that no air may get into it, and ſet it to ſtand in ſome hot place, till all the vinegar is conſumed or diſappears. Laſt of all, draw from the remaining matter the oil by means of an Alembic; which will ſerve to ſeveral purpoſes of Fire-works.

SPORT XLV.

To make Moulds, Rowlers, and Rammers for Rockets of all ſorts.

A Rocket, which the French call *Fuſee*, the Latins *Rochetta*, and the Greeks *Byrobolos*; conſiſts of a Cartouch or paper tube called the Coffin, and a combuſtible compoſition, with which it is loaded; which being fired, mounts into the air, in a manner moſt agreeable to behold.

There are three ſorts of them; the Small, the Middling and the Great. All ſuch are reckoned Small, whereof the diameters do not exceed that of a lead bullet of one pound, or whoſe moulds admit not a bullet above that weight. The Middling are thoſe the moulds of which will admit bullets from one to three pounds weight. The Great will carry from a three pound to an hundred pound ball.

To determine the bigneſs of theſe Coffins to a required meaſure, that is, length and thickneſs, and to make any demanded number of them, of the ſame reach, and of equal force, they muſt be fitted to a concave cylinder, made of ſome hard matter, and turned exactly in a lathe: this is called the Mould or Form, which is ſometimes made of metal, but moſt commonly of hard wood, ſuch as Box, Juniper, Aſh, Cypreſs, wild Plum-tree, Italian Walnut-tree, and ſuch like.

Beſides this, there is another but a convex and ſolid cylinder of wood required, called a Rowler, upon which the thick paper, whereof the Coffin is made, muſt be rowled, till it is of a bigneſs exactly to fill the concavity of the Mould. The diameter of this Rowler muſt contain five eighth parts of that of the Mould, the length of which muſt be ſix times the diameter of its bore, in ſmall Rockets; but in the middling and the large ones it muſt be only five, or four times the length of the diameter of their bore.

Another cylinder of wood muſt alſo be had, which is to be a little ſmaller than the former, that it may go into the Coffin with the greater eaſe. And this is to ſerve for a Rammer, to drive down the Compoſition into the Coffin when you charge it. But firſt your Coffin muſt be ſtraitened or choaked, which is done by winding a cord about the end of it, after you have a little withdrawn the Rowler, turning in the mean time the Coffin, and drawing the cord, till there remains only a little hole, which then muſt be tied with ſtrong packthread. This done, you muſt draw out the Rowler, and introducing the Rammer into the Coffin, put all into the Mould; and when you have ſtruck five or ſix blows with a mallet upon the Rammer, to give a good form to the neck of the Rocket, the Coffin is finiſhed, and ready to be filled upon occaſion.

This Rammer muſt be bored lengthwiſe to ſome depth, that it may receive into its concavity the Needle which muſt be in the mould, together with the Coffin and Rammer, the uſe of this needle, which muſt be one third part of the length of the Coffin or Mould is to make a vent for the priming in the bottom of the Compoſition, of which we ſpeak in the enſuing Sport.

SPORT XLVI.

To prepare a Compoſition for Rockets of any ſize.

THE Compoſition wherewith the coffins are to be filled is different according to the different bigneſs of them; for it is found by experience, that what is fit for ſmall rockets, burns too violently, and too quickly in thoſe that are large, becauſe the fire is bigger, and the matter alſo driven cloſer together: hence it is that no gun-powder is uſed in the larger ſort. In making up this compoſition, according to the different ſizes of rockets, the following proportions muſt be obſerved.

For Rockets from 60 to 100 pounds, you muſt to three pounds of Saltpetre, add one pound of Sulphur, and two pounds of good Wood-coal.

If they are from 30 to 50 pounds, to 30 pounds of Saltpetre, put ſeven pounds of Sulphur, and ſixteen pounds of Coal.

Rockets from 18 to 20 pounds, to twenty-one pounds of Saltpetre, require ſix of Sulphur and thirteen of Coal.

From 12 to 15 pounds, require to four pound of Saltpetre one pound of Sulphur, and two pounds of Coal.

If they be from 9 to 12 pounds; to ſixty-two pounds of Saltpetre, add 9 pounds of Sulphur, and twenty of Coal.

From 6 to 9 pounds; add to ſeven pounds of Saltpetre, one of Sulphur, and two of Coal.

From 4 to 5 pounds; to eight pounds of Saltpetre, add one pound of Sulphur, and two of Coal.

From 2 to 3 pounds; to ſixty pounds of Saltpetre, add two of Sulphur, and 15 of Coal.

For one pound; to ſixteen pounds of Gunpowder, add one pound of Sulphur, and three of Coal; or to nine ponnds of Powder, four of Saltpetre, one of Sulphur, and two of Coal.

For twelve ounces; put to nine pounds of Powder, four of Saltpetre, one of Sulphur, and two of Coal.

For eight ounces; add to thirty pounds of Powder, twenty-four of Saltpetre, three of Sulphur, and eight of Coal.

For five and ſix ounces; to thirty pounds of Powder, add twenty-four pounds of Saltpetre, three pounds of Sulphur, and eight pounds of Coal.

For four ounces; add to twenty-four pounds of Powder, four pounds of Saltpetre, two pounds of Sulphur, and three pounds of Coal.

For two and three ounces; to twenty-four pounds of Powder, put four pounds of Saltpetre, one pound of Sulphur, and three pounds of Coal.

For an half ounce, and an ounce; take fifteen pounds of Powder and two pounds of Coal.

For the ſmaller Rockets: to nine or ten pounds of Powder, add one pound, or one and a half of Coal.

Here follow alſo other proportions, which experience hath taught to ſucceed extremely well.

For Rockets that contain one or two ounces of matter: Add to one pound of Gun-powder, two ounces of good Coal; or, to one pound of Muſket-powder, take one pound of coarſe Cannon-powder; or, to nine ounces of Muſket-powder, put two ounces of Coal: or to one ounce of Powder, an ounce and a half of Saltpetre, with as much Coal.

For Rockets of two or three ounces; add to four ounces of Powder, one ounce of Coal; or to nine ounces of Powder, two ounces of Saltpetre.

For a Rocket of four ounces; add to four pounds of Powder, one pound of Saltpetre, and four ounces of Coal, and if you pleaſe, half an ounce of Sulphur: or to one pound two ounces and a half of Powder, four ounces of Sulphur, and two ounces of Coal: or to one pound of Powder, four ounces of Saltpetre, and one ounce of Coal; or to ſeven ounces of Powder, four ounces of Saltpetre, and as much Coal: or, add to three ounces and an half of Powder, ten ounces of Saltpetre, and three ounces and an half of Coal. The compoſition will be yet more ſtrong, if it be made up of ten ounces of Powder, three ounces and an half of Saltpetre, and three ounces of Coal.

For Rockets of five or ſix ounces, take two pounds five ounces of Powder, to half a pound of Saltpetre, two ounces of Sulphur, ſix ounces of Coal, and two ounces of Filings of Iron.

For Rockets of ſeven or eight ounces; add to ſeventeen ounces of Powder, four ounces of Saltpetre, and three ounces of Sulphur.

For Rockets from eight to ten ounces; to two pounds five ounces of Powder, put half a pound of Saltpetre, two ounces of Sulphur, ſeven ounces of Coal, and three ounces of Filings.

For Rockets from ten to twelve ounces; take to ſeventeen ounces of Powder, four ounces of Saltpetre, three ounces and an half of Sulphur, and one ounce of Coal.

For Rockets from fourteen to fifteen ounces, to two pounds four ounces of Powder muſt be added, nine ounces of Saltpetre, three ounces of Sulphur, five ounces of Coal, and three ounces of File duſt.

For Rockets of one pound, to one pound of Powder, take one ounce of Sulphur, and three ounces of Coal.

For a Rocket of two pounds, add to one pound four ounces of Powder, twelve ounces of Saltpetre, one ounce of Sulphur, three ounces of Coal, and two ounces of File duſt of Iron.

For a Rocket of three pounds, to thirty ounces of Saltpetre, put ſeven ounces and an half of Sulphur, and eleven ounces of Coal.

For Rockets of four, five, ſix, or ſeven pounds, add to thirty one pounds of Saltpetre, four pounds and an half of Sulphur, and ten pounds of Coal.

For Rockets of eight, nine, or ten pounds, take to eight pounds of Saltpetre, one pound four ounces of Sulphur, and two pounds twelve ounces of Coal.

The proportion of the different materials being thus determined, each of them muſt be well beaten, and ſearced apart, and afterwards weighed and mixed. Thus is your compoſition ready where withal to charge your Coffins, which muſt be made of ſtrong paper well paſted.

SPORT XLVII.

To make a Rocket.

YOUR Coffins and different Compoſitions being in readineſs, you muſt chuſe a compoſition ſuitable to the largeneſs of your deſigned Rocket, which muſt neither be too wet nor too dry, but a little moiſtened with ſome oily liquor, or with brandy; then take your Coffin, the length of which muſt be proportioned to the bigneſs of its concavity; put it, with the Rammer, into the Mould; then put into it ſome of your compoſition, taking good care not to put in too much at a time, but only one ſpoonful or two; then put in your Rammer, and with a Mallet ſuited to the bigneſs of the Coffin, ſtrike three or four ſmart blows directly upon it; then withdraw the Rammer again, and pour in an equal quantity of your compoſition, and drive it down in like manner with your Rammer and Mallet, giving the ſame number of blows; continue thus doing till the coffin is filled to the height of the mould, or rather a little below it, that five or ſix folds of the paper may be doubled down upon the compoſition thus driven into the Coffin, which ſometimes inſtead of paper is made of wood.

The Coffin being filled with the mixture, and the paper doubled down upon it, you muſt beat it hard with the Rammer and Mallet to preſs down the folds of the paper, upon which you may put ſome Corn-powder, that it may give a report. In this paper folded down, you muſt make three or four holes with a bodkin, which muſt penetrate to the compoſition, to ſet fire to the Stars, Serpents, and Ground-rockets, when ſuch there are; otherwiſe it will ſuffice to make one hole only, with a broach or bodkin, which muſt be neither too ſmall nor too great, but about one fourth of the diameter of the bore, as ſtraight as poſſible, and in the very middle, in order to fire the Corn powder.

SPORT XLVIII.

To make Sky-Rockets that mount into the Air, with Sticks.

IT is to be noted, that the head of a Rocket, is the higheſt end, by which it is loaded, and which riſes firſt when it is fired: the neck of the Rocket, or its tail, is the lower end, where it was choaked or ſtraightened, and the priming is put, which muſt be of good Corn-powder.

Your Rocket being charged, as was taught in the preceding Sport, you muſt have a long rod or ſtick, of ſome light wood, ſuch as Oſier, or Fir, which muſt be bigger and flat at one end, growing ſlenderer towards the other. This ſtick muſt be ſtraight and ſmooth, without knots, and plained if need be. Its length and weight muſt be proportioned to the ſize of the Rocket, being ſix, ſeven, or eight times the length of it; to the larger end of this, where it is flatted, you muſt tie your Rocket, its head reaching a little beyond the end of the ſtick, and being thus fixed, lay it upon your finger two or three inches from the neck of the Rocket, which ſhould then be exactly balanced by the ſtick, if it is rightly fitted; after which you have nothing to do, but to hang it looſely upon two nails, perpendicular to the horizon, with its head up, and then it is ready for firing. But if you would have it to riſe very high, and in a ſtraight line, you muſt put a pointed paper cap upon its head, and it will pierce the air with greater facility.

To theſe Rockets, for the greater diverſion of the ſpectators, ſeveral other things may be added: as Petards or Crackers, thus; get a box of iron ſoldered, of a convenient bigneſs, fill it with fine grain-powder; put it into the Coffin upon the compoſition, with the touch-hole down, double the reſt of the paper upon it to hold it faſt till the mixture is conſumed, and then firing it will give a report in the air.

You may add to them likewiſe, Stars, Golden-rain, Serpents, Fire-links, and other ſuch agreeable works, the making of which ſhall be taught afterwards. In order to this, you muſt have in readineſs an empty Coffin, of a larger diameter than your Rocket. This muſt be choaked at one end, ſo as only to admit the head of the Rocket, to which it muſt be faſtened. Into this large Coffin, having firſt ſtrewed the bottom of it with Meal-powder, you muſt put your Serpents, or Golden-rain, or Fire-links, with the primed end downwards; and amongſt, and over your ſtars you muſt throw a little powder. Then you may cover this additional Coffin with a piece of paper, and fit to it a pointed cap as before, to facilitate its aſcention.

SPORT XLIX.

To make Sky-Rockets which riſe into the air without a ſtick.

SKY-ROCKETS without ſticks muſt be ſmall, becauſe they are held in the hand from whence they riſe, after you have put fire to the priming they are made as the foregoing; but that they may the better fly into the air, you muſt fit to them four wings diſpoſed croſs-wiſe, like the feathers of darts or arrows; their length muſt be one-third part of that of the rocket, their breadth at the lower part half their length, and their thickneſs about a ſixth or eight part of the diameter of the orifice of the rocket.

Inſtead of four of theſe wings, you may uſe three of the ſame dimenſions with equal ſucceſs; but with this caution, that in placing them upon your rocket, the lower ends of them muſt be let down below the tail of it the length of one diameter of its orifice. There are many other ways of making theſe rockets, according to the various fancies of artiſts, which would be too tedious for this work.

If the compoſition for your Rockets is defective, as is known when they riſe, either not at all, or with difficulty, or fall down again before conſumption of the mixture; or when they mount not with an equal and upright motion, but turning and winding, or whirling in the air; to amend your compoſition, you muſt diminiſh the quantity of coal when it is too weak, and add to it if too ſtrong, as it is when it burſts the rocket, the coal ſerving to abate the force of the powder, and to give a fine train to your rocket. Wherefore it would be convenient, before you make up a quantity of rockets, to try your mixture and correct its faults.

To preſerve your rockets in good condition, they muſt be kept in a place neither too dry, nor too moiſt, but temperate; and the compoſition ſhould not be made up, but upon occaſion to uſe it. Your rocket muſt not be pierced till you deſign to play it; which muſt not be in a ſeaſon of wind or rain, or when the nights are moiſt with fogs and miſts, all which are prejudicial to the agreeable effects of a rocket.

If you would have your rocket to burn with a pale white flame, mix ſome Camphire with your compoſition; inſtead of which if you take raſpings of Ivory, the flame will be of a clear ſilver-colour, but ſomewhat inclining to that of lead; if Colophony or Grecian pitch, it will be of a reddiſh copper-colour; if black, or common pitch, the flame will be dark and gloomy; if Sulphur, it will be blue; if Sal-armoniack, it will appear greeniſh; if crude Antimony, or the raſpings of yellow Amber, it will emit flames of a like colour.

SPORT L.

To make Ground Rockets, which run upon the earth.

ROCKETS that run along the Ground, called therefore, Ground-Rockets, require not ſo ſtrong a compoſition, as thoſe that mount into the air; and therefore continue longer burning as well as moving more ſlowly: Wherefore they vary from the others, as well in the dimenſions of their Coffins, as in the compoſition wherewith theſe are charged. The length of the bore or concavity, may be eleven times that of its diameter; the rowler on which the coffin is made, may be five lines in diameter, and the rammer a little leſs, that it may go eaſily into the coffin without ſpoiling it.

The compoſition may be of Cannon-powder only, well beaten and ſearced till it is as fine as flower, wherewith you muſt fill the coffin, by little and little as before, within a finger's breadth of the brim of the mould; then doubling down one-third part of the paper, knock it down with the rammer and mallet, and after with a bodkin, make a ſmall hole which may penetrate to the compoſition; then put in a piſtol-charge of fine powder, doubling down ſome more of the paper upon it, the reſt of which muſt be choak'd tying it hard with pack-thread.

Theſe rockets being ſmall, are charged only with powder finely pulveriſed, without any coal, herein differing from the large ones, that have no powder at all, except in their priming which in both ſorts muſt be of well grained powder: The reaſon of which is, becauſe in a greater concavity there is a greater fire acting upon a greater quantity of matter, and conſequently with more violence; there being alſo a greater quantity of air to be rarified in a great than in a ſmall rocket.

When you choak or ſtreighten the end of your Rocket, whether ſmall or great, you muſt have a hook or ſtaple driven into a poſt, or into a wall, to this tie one end of your cord, which muſt be of a ſize proportionable to your rocket, or to the bar of a window, and the other to a ſtrong ſtick, which you muſt put between your legs: thus the cord being winded about your rocket in the deſigned place, you may draw, turning and ſtraightening it by degrees as you deſire.

SPORT LI.

To make Rockets that fly on a line, called Air-Rockets.

THIS is done with ordinary rockets that muſt not be too big, by faſtening to them two iron rings, or, which in my opinion is better, a wooden pipe or cane, through which muſt paſs a well ſtretched line: thus if you ſet fire to your rocket, it will run along the line without ceaſing till all the matter is ſpent.

If you would have your rocket to run back, as well as forward, after you have filled one half of the coffin with the compoſition, ſeparate this from the empty half by a wheel of wood fitted exactly to the cavity; in the middle of this wheel muſt be a hole, from which a ſmall pipe, filled with meal-powder, muſt paſs along the middle of the empty half, which then muſt be filled with the compoſition; and ſo after the firſt half of the rocket is conſumed, the fire being communicated by the little pipe, will light it at the other extremity, and ſo drive it back to the place from whence it came.

The ſame thing may be effected by the means of two rockets tied together, the tail of the one to the head of the other, one of which being burnt to the end fires the other, making it to run back: but leaſt the ſecond ſhould catch fire at the head, it muſt be defended with a cover of paper or waxed cloth.

This ſort of rockets is commonly uſed to ſet fire to other machines in fire works for diverſion, to which, for the greater pleaſure, they give the figures of ſeveral animals, ſuch as ſerpents or dragons, which then are called Flying Dragons; and are extremely agreeable, chiefly when filled with ſeveral other works, as golden rain, hairs dipt in wild fire, ſmall nut-ſhells filled with the rocket compoſition, and many other diverting things, of which afterwards.

SPORT LII.

To make Rockets that burn in the water, called Water-Rockets.

THOUGH the fire and water are oppoſite elements mutually deſtroying one another, yet the rockets we have hitherto deſcribed, being once lighted will continue to burn even in the water, and will have their full effect; but for as much as it is done under water, we are deprived of the pleaſure of beholding it. In order therefore, to make them to ſwim upon the water, we muſt alter ſomewhat the proportions of their mould, as well as the materials of their compoſition.

The mould, then, required to ſuch rockets, may be eight inches in length, and its bore an inch over. The rowler muſt be of nine lines diameter, and the rammer not quite ſo thick: no needle is required to this mould.

The compoſition, if you would have your rocket burn on the water, with a clear flame like a candle, muſt be made of three ounces of powder beaten and ſearced, one pound of ſaltpetre, and eight ounces of ſulphur mixed together: when you deſire your rocket to appear on the water with a fine tail, you muſt, to eight ounces of common powder, add one pound of ſaltpetre, eight ounces of ſulphur, and two ounces of coal.

The compoſition being prepared, and the coffin charged with it, as is taught above, put a fire-link at the end of it; and covering your rocket with wax, pitch, or roſin, to preſerve the paper from the water, faſten to it a ſtick of white willow about two foot long, which will cauſe it to ſwim upon the water.

Many other different ways may ſuch rockets be made without altering either the mould or compoſition; for which the curious may conſult the authors that have writ particular treatiſes of Pyrotechny.

A rocket alſo may be made, which, after burning ſome time in the water, will throw up into the air ſparkles and ſtars; which is done by dividing the rocket into two parts with a wooden wheel having a hole in the middle, one partition being filled with the common compoſition, the other with ſtars, having ſome powder ſtrewed amongſt them.

Moreover you may contrive a rocket, which having burnt one half of its time in the water, will mount up in the air with great ſwiftneſs; thus: having filled two equal coffins with good compoſition, paſt them together ſlightly only at the middle, the head of the one anſwering the tail of the other; betwixt them muſt paſs a little pipe at the extremity, to light the other when one is conſumed. Then faſten the rocket to which the other is joined, to a ſtick of ſuch length and bigneſs as is required for ballancing it, and to the lower end of the rocket tie a pack-thread, to which you muſt faſten a large muſket-ball that muſt hang upon the ſtick by means of a bent wire. This done, ſet fire to your rocket, it being in the water: and its compoſition being conſumed, will light, by means of a little pipe, the other rocket, which will mount into the air, through the ſtrength of the fire, the firſt being kept down by the weight it ſuſtains.

SPORT LIII.

To make Fire Links.

A *Fire-Link*, ſo called from its reſemblance, to the links of a ſaucidge, is a kind of Rocket, that is uſually tied to the end of a bigger one, to render the effect more agreeable. I ſaid, uſually, becauſe there are ſome of them made that fly into the air as Sky-rockets, and are called Flying Fire-links, to diſtinguiſh them from the others which are named fixed Fire-links. We ſhall here briefly teach the making of both ſorts.

And firſt the fixed kind to be faſtened to a Rocket is made thus: Take a coffin of what bigneſs yon think fit, and having choaked it at the end, fill it with fine powder, and choak it at the other end: then roll it ſtrongly with ſmall cord from one end to the other, gluing the cord with good glue, to keep it faſt, and to ſtrengthen the coffin, that it may give the greater noiſe when it breaks: thus is your fire-link ready to be faſtened to the end of a Rocket, either with paper, parchment, or cord, or otherwiſe; but note, that you muſt pierce the end of your Fire-link, which joins to the Rocket, and prime it with Corn-powder.

To make flying Fire links, you muſt have ſuch coffins as for the former, only they muſt be a little longer, and having choaked them at one end, charge them with Corn powder, adding at leaſt meal-powder to the thickneſs of one inch, driving all down, as in Sky-rockets, with a mallet. Then ſtrengthen the coffin with a line, as in the former, after you have choaked the other end, leaving a hole about the bigneſs of a gooſe quill, to which you muſt put a little moiſtened powder for priming.

Or, having choaked at one end, and charged your coffin, within one inch of the other end, choak it there, leaving only a ſmall hole, which if quite ſhut up, or too ſmall, muſt be opened with a bodkin; then fill up your empty ſpace with powder finely flowered, or with the compoſition for Sky-rockets, which muſt be driven cloſe with a rammer and mallet, doubling down the remaining paper, if any, upon your compoſition, which will give a fine tail to your link; and when you have made a hole in the middle of this laſt paper, and primed it, your flying link is ready to be thrown into the air, which is done thus:

You muſt provide guns or cannons with a vent at bottom, where there muſt be a tail ſomewhat long, which muſt paſs through a piece of wood that it may reach to a fire conveyance running along underneath, to ſet fire to the cannons one after another, which will alſo throw up into the air the links with a noiſe in the ſame order.

SPORT LIV.

To make Fire-Lances.

LANCES of fire, are long and thick pipes or cannons of wood, with handles at the end, whereby they are made faſt to ſtakes or poſts, well fixed, that may ſuſtain the force of the fire, having ſeveral holes to contain rockets or petards. They are uſed in feſtival fireworks that repreſent nocturnal fights, as well for throwing rockets, as making vollies of reports.

You muſt uſe them thus: Put a rocket into every hole, and fill the bore of the cannon with compoſition, which fired will, as it conſumes, fire the rockets one after another, and throw them up into the air. But if you would have many thrown up at once, cover the bottom of the lance with compoſition, and thereupon place a long ſmall pipe filled with the ſame compoſition, about which put your rockets, till you have filled your cannon, the primed end being downwards, that ſo firing the compoſition in the pipe, this may light that at the bottom of the lance, which firing the rockets, they will mount all at once into the air.

There may be many other ways of contriving Fire-Lances in imitation of this, of which I ſhall not ſpeak. I ſhall only mention one other ſort of theſe lances. This conſiſts of a coffin made of ſtrong paper well glued, which may be of what dimenſions you think fit, according as it is deſigned to give more or leſs light; this muſt be filled with the Star-compoſition, pulveriſed and primed with meal-powder moiſtened: the lower end muſt be ſtopped with a round piece of wood, which muſt appear two inches without the coffin, that thereby it may be faſtened at pleaſure.

Remark. The name of fiery or burning lances and pikes, is alſo given to a kind of pikes like a javelin or dart, with a ſtrong iron-pointed head, called by the Latins *Phalorica*, and *Dardi di Fuoco* by the Italians, which were formerly thrown, being firſt fired againſt the enemies, either by the hand or from engines, being covered between the iron and wood with tow dipt in ſulphur, roſin, Jews' pitch, and boiling oil; where they lighted they ſtuck, ſetting on fire whatever was inflammable.

This ſort of lances is not now in uſe, but inſtead of them we have Burning Arrows, that are not leſs terrible, though not much now in eſteem; however, we will here gratify the curious with a brief deſcription of them. Flaming Arrows are artificial firebrands thrown amongſt the enemies' works, to reduce them to aſhes: they are made thus——Prepare a little bag of ſtrong coarſe cloth, about the bigneſs of a gooſe's or ſwan's egg, of a globular or ſphæriodal figure, which muſt be filled with a compoſition made of four pounds of beaten powder, as much refined ſaltpetre, two pounds of ſulphur, and one pound of Grecian pitch: or you may make it of two pounds of meal-powder, eight pounds of ſaltpetre refined, two pounds of ſulphur, one pound of camphire, and one pound of Colophony: or yet more ſimply thus——of three pounds of powder, four pounds of ſaltpetre, and two pounds of ſulphur. With one of theſe mixtures fill the bag, preſſing it hard, and make an hole through the middle of it lengthwiſe, to receive an arrow, like thoſe of the ordinary bows or croſs-bows, the head of it remaining without the bag, which muſt be faſtened ſo as it may not move or ſlide towards the feathers. This done; roll your bag with ſtrong packthread as thick as poſſible from one end to another, and then cover it all over with mealpowder mixed with melted pitch. Thus it is ready to be ſhot out of a bow or croſs bow, after it is fired by two little holes made for that purpoſe near the head of your arrow.

SPORT LV.

To make Stars for Sky-Rockets.

STARS are little balls about the bigneſs of a muſquet-ball, or an hazle-nut, made of an inflammable compoſition, which gives a ſplendid light, reſembling that of ſtars, from whence is the name. When they are put into the rocket, they muſt be covered with prepared tow, the manner of making which ſhall be taught, after that of ſtars.

They are made thus: to one pound of powder finely flowered, add four pounds of ſaltpetre, and two pounds of ſulphur; and having mixed all very well, roll up about the bigneſs of a nutmeg of this mixture in a piece of old linen or in paper; then tie it well with packthread, and make a hole through the middle with a pretty big bodkin, to receive ſome prepared tow, which will ſerve for priming: this being lighted, fires the compoſition, which emitting a flame through both holes, gives the reſemblance of a pretty large ſtar.

If inſtead of a dry compoſition, you uſe a moiſt one in form of paſte, you need only roll it into a little ball, without wrapping it up in any thing, ſave, if you will, in prepared tow, becauſe of itſelf it will preſerve its ſpherical figure; nor needs there any priming, becauſe while moiſt you may roll it in meal-powder, which will ſtick to it, and when fired will light the compoſition, and this at falling forms itſelf into drops.

Remark. There are many other ways of making ſtars, too long now to be mentioned; I ſhall only here ſhew how to make Stars of Report, that is, ſtars that give a crack like that of a piſtol or muſquet, as follows:

Take ſmall links, made as is taught in Sport 53, which you may chooſe either to roll with line or not, tie to one end of them, which muſt be pierced, your ſtars if made after the firſt manner, that is, of the dry compoſition: otherwiſe you need only leave a little piece of the coffin empty beyond the choak of the pierced end, to be filled with moiſt compoſition, having firſt primed your vent with grain powder.

You may alſo contrive ſtars, which, upon conſumption of the compoſition, may appear to be turned into ſerpents, a thing eaſy to be performed by ſuch as underſtand what precedes; upon which account, and becauſe they are but little in uſe, I ſhall ſa no more of them.

SPORT LVI.

To make prepared Tow for priming to Fire-works

PREPARED Tow, called alſo Pyrotechnical match, and Quick-match, to diſtinguiſh it from common match, is uſed for priming all ſorts of machines for Fire-works of diverſion, ſuch as rockets, fire-lances, ſtars, and the like; and it is made as follows:

Take thread of flax, hemp or cotton, and double it eight or nine times, if it is for priming your large rockets, or fiery lances; but four or five times only, if it is to be put thro' your ſtars. Having made it of a bigneſs proportioned to your deſigned uſe, and twiſted it, but not too hard, wet it in clean water, which muſt be after ſqueezed out with your hands. Then put ſome gun-powder in a little water, ſo as to thicken it a little; in this ſoak your match well, turning and ſtirring it till it is thoroughly impregnated with the powder; and then taking it out, roll it in ſome good powder duſt, and hang it upon lines to dry either in the ſun or ſhade: thus you have a Pyrotechnical match ready for uſe on all occaſions.

Common Match, called alſo Fire cord, is thus made: Take an unglazed earthen pot, cover its bottom with red ſand well waſhed and dried; upon this lay ſpiral-wiſe plain match of cotton, or well cleaned tow, half an inch thick the diſtance of half an inch being between each revolution, and then cover it with ſand; upon which again place a lay of match as before, and upon this another of ſand, and ſo interchangeably till the pot is full, but finiſhing always with a lay of ſand: then cover it with an earthen cover, and lute with clay the joining, ſo as no air may get entrance. This done put burning coals round the pot, and after it has been kept hot for ſome hours, let it cool of itſelf; ſo your match is prepared, which will burn without ſmoak or offenſive ſmell.

SPORT LVII.

To make Golden rain for Sky-rockets.

THERE are ſome Sky-rockets, which in falling make little waves in the air, like unto hair half curled, and are therefore called Hairy-rockets; they end in a ſort of rain of fire, called Golden rain.—'Tis thus made.

Fill with the compoſition for ſky-rockets gooſe quills, the feathers being cut off; putting ſome wet powder in the open end of each, both to keep in the compoſition, and to ſerve for priming: with theſe fill the head of your ſky-rocket, and it will end in a golden rain, very agreeable to behold.

Remark. This golden rain calls to my mind a Pyrotechical-hail, ſo called from its reſemblance to the natural, which is a quantity of ſmall hard bodies, being either pieces of flint, round ſtones, leaden bullets, or ſquare pieces of iron, incloſed in a cartridge of wood, iron, or copper, and is therefore called cartridge or caſe-ſhot; they are uſed in war, either in open field to diſorder an enemy's army, or in a ſiege to drive them away from a breach or gate to be ſeized, being ſhot either out of a mortar, or a great-gun of a large bore.

SPORT LVIII.

To repreſent with Rockets, ſeveral figures in the air.

IF you take a rocket of the larger ſort, and place round the head of it many ſmall ones, fixing their ſticks all round the large coffin upon the head of your big rocket, which uſes to contain the head works, ordering it ſo, that your ſmall rockets take fire, whilſt the great one is mounting up, you will have the reſemblance of a tree, very delightful to the ſight; whereof the big one will repreſent the trunk, and the little ones the branches.

But if the ſmall rockets take fire when the great one is half turned in the air, they will have the appearance of a comet: and when the large one is altogether turned, ſo that its head points downwards to the earth, they will exhibit the ſimilitude of a fountain of fire.

If you put on the head of a large rocket many gooſe-quills, the feathers being cut off, filled with ſky-rocket compoſition, as in the preceding ſport; when fired, they will appear to thoſe under them as a fine ſhower of fire; but to thoſe who view them on one ſide, like half curled hair, very delightful to the view.

Finally, with ſerpents tied to a rocket with packthread by the ends which are not fired, leaving two or three inches of the thread between each, you may repreſent at pleaſure ſeveral ſorts of figures moſt entertaining and agreeable to the ſight.

SPORT LIX.

To make Fire-pots for Fire-works of diverſion.

A Pot of fire is a large Coffin filled with Rockets that take fire all together, and are diſcharged from the pot without hurting it. The bottom of the pot muſt be covered with powder-duſt, which being fired by a match that muſt paſs through a hole in the middle of the pot, will ſet fire to all the rockets at once.

When there are many Fire-pots, they muſt be covered with ſingle paper, that they may not play all at once; otherways, one when fired might ſet fire to another; and you muſt uſe only a ſingle leaf of paper, that it may not hinder the rockets to fly out. Pots of fire are alſo made for war-ſervice.

SPORT LX.

To make Shining-balls for diverſion, and for ſervice in war.

FIRST, to make ſhining-balls for recreation; to four pounds of ſaltpetre, put ſix pounds of ſulphur, two pounds of crude antimony, four pounds of colophony, and four pounds of coal: or, to two pounds of ſaltpetre, take one pound of ſulphur, as much antimony, two pounds of colophony, as much coal, and one pound of black pitch; melt theſe, being well beaten, in a kettle, or in a glazed earthen pot, and thereinto throw ſuch a quantity of hards of flax or of hemp, as will juſt ſuffice to imbibe all the liquor, of which as it cools make little pellets or round balls, to be covered over with prepared tow, which I taught to make in Sport 56, and after put into ſky-rockets, or balls for diverſion, as is uſual to be done with fiery ſtars.

Next, to make ſhining or flaming balls for ſervice in war, to be thrown from a mortar, againſt the enemy, you muſt melt in a kettle, or glazed earthen pot, as above, equal parts of ſulphur, black pitch, roſin and turpentine, into which dip an iron, or ſtone bullet, ſomewhat lower than the bore of the mortar, and when its ſurface is covered with this matter, roll it in corn-powder: which done, cover it over with callico, and dip it again into the ſame liquor; rolling it after in grain-powder; this muſt be reiterated ſeveral times, covering, dipping and rowling it, till it fills exactly the bore of the mortar or cannon, into which you deſign to put it, remembering ſtill to end your operations with rolling it in grain-powder, that being put into your piece, immediately above the charge of the powder, it may take fire as it is thrown into the air againſt the enemy, either to annoy them, or to diſcover their deſigns, which is uſually done in ſieges.

Inſtead of theſe ſhining-balls, red-hot-balls are more frequently uſed for offending the enemy, by burning them, their houſes or works. Theſe bullets are of iron, and being hated red hot in a furnace are thus uſed. Your cannon being charged with powder, freed from corns, and pointed ſomething upwards, you muſt have in readineſs a cylinder of wood fitted exactly to its bore, which you muſt put into your gun next the powder, and upon it you muſt ram down a wad of wet ſtraw, hay, tow or hemp, or ſome ſuch moiſt materials; then putting in your red-hot ball with a ladle, immediately put fire to your gun.

SPORT LXI.

To make a Wheel of Fire-works.

A Wheel of Fire, or Fire-works, is a wheel of light wood, ſet round with rockets of a mid-ſize, the head of one regarding the tail of another, that when the firſt is ſpent, it may ſet fire to the next, which makes the wheel turn round its fixed axel-tree without intermiſſion, till all the rockets are conſumed.

Upon this account it is called a Fire-wheel, and it is alſo called a *Fiery Sun*, becauſe placed horizontally upon a ſtake ſomewhat large and perpendicular to the horizon, it turns round, and repreſents a ſun in night combats, which is very diverting.

You may alſo make fire-wheels which have a ſituation perpendicular to the horizon, and turn upon an axis parallel to it, very agreeable to behold. Fire-wheels are likewiſe uſed to light other works at a diſtance, in aſcending or deſcending upon a ſtretched rope, like flying dragons; and on many other occaſions, to the pleaſure of the ſpectators.

SPORT LXII.

To make a Balloon, or fiery Foot-ball.

BALLOONS are coffins of a large diameter ſhot out of a mortar whither one pleaſes, filled commonly with Serpents about the thickneſs of a Ground-rocket, but not ſo long, with two ſmall fire-links of the ſame length and breadth, which being fired by their priming, burſt the coffin, this having below a fire-conveyance, at the mouth of which there is a priming of cotton dipt in powder.

The coffin is made with a thick wooden rowler, about which is rolled ſtrong card-paper glued, to keep it from undoing, which being choaked below, a hole is made there for a fire-conveyance, filled with a compoſition more ſlow than that of Ground-rockets, being like to that of Sky-rockets: after this it may be filled with Serpents, and ſometimes with Stars, and then choaked above.

SPORT LXIII.

To make an Ointment excellent for curing all ſorts of burnings.

BOIL, over a gentle fire, in common water Hogs' lard, or the fat of freſh pork, ſkimming it perpetually till no further ſcum ariſes; then expoſe it thus melted to cool in the clear open air three or four nights. After this melt the ſame lard or greaſe in an earthen veſſel over a ſlow fire, and ſtrain it through a linen cloth into cold water, and after waſh it well in fair river or fountain water, to take away its ſalt, which will make it become white as ſnow. Finally, being thus purified, put it up in a glazed earthen veſſel, to be kept for uſe upou occaſion.

If it falls out, as commonly it happens, that by a burning, bliſters riſe upon the ſkin, they muſt not be cut or broken, till the ointment has been uſed to it for three or four days. You may alſo uſe the following, which you will find to be of great efficacy, and is made of Hogs'-lard melted and mixed with two drachms of the water of Night ſhade, and one drachm of oil of

Saturn; or with two ounces of juice of Onions, and one ounce of oil of Walnuts.

PART III.

Miſcellaneous SPORTS.

SPORT LXIV.

To deſcribe an artificial Lantern, by which one may read at night at a great diſtance.

MAKE a lantern in the form of a cylinder or of a ſmall caſk laid on one ſide; put in one of its two ends a concave parabolic glaſs, in order to apply to its focus the flame of a wax candle, the light of which will reflect to a great diſtance in paſſing through the other end that ought to be open and will appear with ſuch a ſplendor, that by it one may read at night very ſmall letters at a great diſtance, with teleſcopes; and thoſe who ſee the light of the candle at a great diſtance, will take it to be a great fire, which will be ſtill the lighter if the lantern is tinned within, and made in the form of an elypſis.

The

The Magical Lantern.

We likewiſe make uſe of ſuch a glaſs for a magical lantern, ſo called, becauſe by means of it we can make any thing appear on the white wall of a dark room; ſuch as monſters and fearful apparitions, which the ignorant impute to magic. The light reflected by virtue of this glaſs paſſes through a hole in the lantern, in which there is a lens of glaſs; and between them there is a thin piece of wood containing ſeveral glaſſes painted with monſtrous and formidable figures, which they move up and down through a ſlit in the body of the lantern, and which caſt their repreſentation to any oppoſite wall in the ſame colours and proportions, but much enlarged.

SPORT LXV.

By the means of two plain Looking-glaſſes to make a Face appear under different forms.

HAVING placed one of the two glaſſes horizontally, raiſe the other to about right angle, over the firſt; and while the two glaſſes continue in this poſture, if you come up to the perpendicular glaſs, you will ſee your face quite deformed and imperfect; for it will appear without forehead, eyes, noſe or ears, and nothing will be ſeen but a mouth and a chin raiſed bold. Do but incline the glaſs never ſo little from the perpendicular, and your face will appear with all its parts excepting the eyes and the forehead. Stoop it a little more, and you will ſee two noſes and four eyes; and then a little further, and you will ſee three noſes and ſix eyes. Continue to incline it ſtill a little more, and you will ſee nothing but two noſes, two mouths and two chins; and then a little further again and you will ſee one noſe, and one mouth. At laſt incline a little further, that is, till the angle of inclination comes to be forty-four degrees, and your face will quite diſappear.

If you incline the two glaſſes the one towards the other, you will ſee your faces perfect and intire; and by the different inclinations, you will ſee the repreſentation of your face, upright and inverted alternately, &c.

SPORT LXVI.

To deſcribe a vertical dial upon a pane of glaſs, ſo as to denote the hours without a gnomon.

I Once made ſuch a dial for a friend after the following manner:

I took off a pane of glaſs that was ſoldered on the out-ſide to the frame of a window, and calculating the thickneſs of the frame for the gnomon, had the pane glewed on again to the inſide of the frame, allotting to the meridian line a ſituation perpendicular to the horizon, as it ſhould be in vertical dials, and on the out-ſide, I cauſed to be glewed to the frame oppoſite to the dial, a ſtrong piece of paper unoiled, that ſo the rays of the ſun might penetrate it the leſs, and keep the ſurface of the dial darker. Then to diſtinguiſh the hours without a ſtyle, I made a little hole in the paper with a pin, over againſt the foot of the ſtyle marked upon the dial; and thus the hole repreſenting the tip or end of the ſtyle, and the rays of the ſun paſſing through it, caſt upon the glaſs a ſmall light, that pointed out the hours very prettily in the obſcurity of the dial.

SPORT LXVII.

To find in all parts and at all times, the four cardinal points of the world, without ſeeing the ſun, or the ſtars, or making uſe of a compaſs.

THE four cardinal parts of the world, *viz.* the eaſt, the weſt, the ſouth and the north, are eaſily found by a compaſs, the needle of which being touched with a loadſtone, turns always one of its points towards the ſouth and the other towards the north, which is enough to direct us to eaſt and weſt; for when one ſets his face to the north, the eaſt is on his right, and the weſt is on his left hand.

The north is eaſily diſtinguiſhed in the night by the ſtars, particularly by minding the polar ſtar, which is but two degrees diſtant from the arctic pole; and in the day-time aſtronomers mark the meridian line upon an horizontal plain, by means of the two points of a ſhadow marked before and after noon upon the circumference of a circle deſcribed from the point of the ſtylus, the ſhadow of which is made uſe of to ſhew by its extremity upon that circumference two points equally remote from the meridian.

But without all theſe helps you may at all times, and in all parts, find out the meridian line, after the following manner.

Take a platter or baſon full of water, and when the water is ſettled and ſtill, put ſoftly into it an iron or ſteel needle, ſuch as a common ſewing needle; and if the needle is dry, and be laid all along upon the ſurface of the water, it will not ſink; but after ſeveral turns it will ſtop in the plan of the meridian circle, ſo that it repreſents the meridian line; and by conſequence one end of it will point to the ſouth, and the other to the north: but without ſeeing the ſun or the ſtars, it is not eaſy to know which of the two ends points to the ſouth and which to the north.

Father Kircher lays down an eaſy way of knowing ſouth and north. He orders you to cut horizontally a very ſtraight tree growing in the middle of a plain at a diſtance from any eminence or wall that may ſhelter it from the wind or the rays of the ſun: in the ſection of that trunk you will find ſeveral curve lines round the ſap which lie cloſer on one ſide than the other.

And, as he ſays, the north lies on that ſide where the lines are moſt contracted, perhaps becauſe the cold ariſing from the north binds up, and the heat from the ſouth ſpreads and rarifies the humours and matter, of which theſe crooked lines are formed. Theſe lines, ſays that author, are as the circumferences of concentrical circles, in ebony or Brazil wood.

SPORT LXVIII.

To break with a ſtick another ſtick reſting upon two glaſſes without breaking the glaſſes.

THE ſtick that is to be broken muſt not be very thick, nor yet lean much upon the glaſſes; it ought, as near as poſſible, to be equally thick all over, for the eaſier finding of its center of gravity, which will then be in the middle.

The ſtick being thus qualified, we lay its two ends, which ought to terminate in a point, upon the brim or edge of two glaſſes of equal height, ſo that the ſtick does not lean to one ſide or end more than the other, and the two points ends reſt but lightly upon the edge of each glaſs, to the end that when it bends a little thro' the violence of the ſtroke, it may eaſily ſlip off, and break at the ſame time. This done, we take another ſtick, and with that give a ſmart blow upon the midling point, which being the centre of gravity will receive all the force of the blow; thus will the ſtick break, and that the more eaſily that the blow is violent, and fall clear of the two glaſſes which remain unbroken, becauſe the ſtick lay but very gently and equally upon the brim of each; for if it reſts more upon one glaſs than the other, it will preſs that one moſt, and ſo may break it.

SPORT LXIX.

To make a lamp fit to carry in one's pocket, that ſhall not go out though you roll it upon the ground.

TO make a lamp that never ſpills its oil, and never goes out in any poſition whatſoever, make faſt the veſſel that contains the oil and the match, to an iron or braſs ring, with two ſmall pivots or hinges diametrically oppoſite, that ſo the veſſel may by its weight continue in æquilibrio round the two hinges and turn with freedom within the circle, ſo as to keep always to an horizontal poſition, as in your ſea compaſſes, which have two ſuch circles to keep them horizontally: and in like manner this firſt circle ought to have two other pivots diametrically oppoſite, which enter into another circle of the ſame ſubſtance; and that ſecond circle has two other little hinges inſerted in another concave body that ſurrounds the whole lamp. Thus the lamp with its two circles may turn freely upon its ſix hinges, which give to the lamp when it is turned, ſix different poſitions, *viz.* up and down, forwards and backwards, to the right and left, and which ſerve to keep the lamp in an horizontal poſition, which, being in the middle does always reſt upon on its centre of gravity, that is, its center of gravity is always in the line of direction which hinders the oil to ſpill, turn it which way you will.

SPORT LXX.

To take up a boat that is ſunk with a cargo of goods.

IF a boat ſinks in a deep river, you may bring her up again, by getting two other boats, one empty, and the other deep loaded with ſome heavy ſubſtance, as ſtones, *&c.* You muſt tie theſe two boats to the boat that is ſunk with two ropes, and extending the rope of the deep laden boat, unload her into the other that is empty; which will raiſe the firſt boat a little, and make it draw along with it the boat that is under water, and at the ſome time make the ſecond boat ſwim ſo much deeper in the water. The ſecond boat being thus loaded, you muſt bend her roap and unload her again into the empty boat, and thereupon ſhe becoming lighter, will riſe and draw the boat under water ſo far further up. Thus you continue to load and unload till you bring the boat even with the water, and then row her to the ſide.

SPORT LXXI.

To make a boat go of itſelf up a rapid current.

THE more rapid a river is, the eaſier it is to make a boat go of itſelf up againſt the current, by a rope and a wheet with its axeltree that has wings like the wings or ſweeps of a mill-wheel.

Fix the wheel with its axeltree at the place to which you would have the boat conducted, and let its ſweeps be as deep in the water as there is occaſion for turning it round: tie a roap to the boat, and to the axeltree of the wheel, which turning with its axeltree by virtue of the rapidity of the water, will wind up the rope on its axeltree, and ſo by the ſucceſſive abbreviation of the rope, drag it againſt the current to the place propoſed: which it will reach ſo much the ſooner that the current is rapid, the rapidity quickening the motion of the wheel.

SPORT LXXII.

To know which of two different waters is the lighteſt, without any ſcales.

TAKE a ſolid body, the ſpecific gravity of which is leſs than that of warer, dale or firwood, for inſtance; and put it into each of the two waters, and reſt aſſured that it will ſink deeper in the lighter than in the heavier water; and ſo by obſerving the difference of the ſinking you will know which is the lighteſt water, and conſequently the wholſomeſt for drinking.

SPORT LXXIII.

To know if a ſuſpicious piece of Money is good or bad.

IF it be a piece of ſilver that is not very thick, as a crown or half crown, the goodneſs of which you want to try; take another piece of good ſilver of equal ballance with it, and tie both pieces with thread or horſe-hair to the ſcales of an exact ballance (to avoid the wetting of the ſcales themſelves) and dip the two pieces thus tied in water; for then if they are of equal goodneſs, that is, of equal purity, they will hang in æquilibrio in the water as well as in the air; but if the piece in queſtion is lighter in the water than in the other, it is certainly falſe, that is, there is ſome other metal mixed with it that has leſs ſpecific gravity than ſilver, ſuch as copper; if it is heavier than the other, it is likewiſe bad, as being mixed with a metal of greater ſpecific gravity than ſilver, ſuch as lead.

If the piece propoſed is very thick, ſuch as that crown of gold that Hiero king of Syracuſa ſent to Archimedes to know if the goldſmith had put into it all the 18 pounds of gold that he had given him for that end; take a piece of pure gold of equal weight with the crown propoſed, *viz.* 18 pounds, and without taking the trouble of weighing them in water, put them into a veſſel full of water, one after another, and that which drives out moſt water, muſt neceſſarily be mixed with another metal of leſs ſpecific gravity than gold, as taking up more ſpace though of equal weight.

SPORT LXXIV.

To know how the wind ſtands, without ſtirring out of one's chamber.

FIX to the cieling of your room a circle divided into 32 equal parts, with the names of the 32 rumps or wind-points, the points of north and ſouth being upon the meridian line. The circles or dial thus divided, muſt have a needle or hand moveable round its centre, like the hand of a watch or clock; and that hand muſt be fixed to an axeltree that is perpendicular to the horizon, and may move eaſily upon the leaſt wind, by virtue of a fane on its upper end above the roof of the houſe, and then the wind turning the fane, will at the ſame time turn its axeltree, and the hand that is fixed to it, which will accordingly point to the rump from whence the wind blows.

SPORT LXXV.

When two veſſels or cheſts are like one another, and of equal weight, being filled with different metals, to diſtinguiſh the one from the other.

THIS Sport is eaſily performed, if we conſider that two pieces of different metals of equal weight in air, do not weigh equally in water; becauſe that of greateſt ſpecific gravity takes up a leſſer ſpace in water, it being a certain truth, that any metal weighs leſs in water than in air, by reaſon of the water, the room of which it fills. For example, if the water weighs a pound, the metal will weigh in that water a pound leſs than in the air. This gravitation diminiſhes more or leſs according as the ſpecific gravity of the metal is greater than that of the water.

We will ſuppoſe then two cheſts perfectly like one another, of equal weight in the air, one of which is full of gold, and the other of ſilver; we weigh them in water, and that which then weighs down the other muſt needs be the gold cheſt, the ſpecific gravity of gold being greater than that of ſilver, which makes the gold loſe leſs of its gravitation in water than the ſilver. We know by experience, that gold loſes in water about an eighteenth part only, whereas ſilver loſes near a tenth part: ſo that if each of the two cheſts weighs in the air, for example, 180 pounds, the cheſt that is full of gold will loſe in the water ten pounds of its weight; and the cheſt that is full of ſilver will loſe eighteen; that is, the cheſt full of gold will weigh 170 pounds, and that of ſilver only 162.

Or, if you will, conſider that gold is of a greater ſpecific gravity than ſilver, the cheſt full of gold, though ſimilar and of equal weight with the other, muſt needs have a leſſer bulk than the other. And therefore if you dip ſeparately each of them into a veſſel full of water, you may conclude that the cheſt which expels leſs water has the leſſer bulk, and conſequently contains the gold.

SPORT LXXVI.

To meaſure the depth of the ſea.

TIE a great weight to a very long cord, or rope; and let it fall into the ſea till you find it can deſcend no farther, which will happen when the weight touches the bottom of the ſea, if the quantity or bulk of water the room of which is taken up by the weight and the rope, weighs leſs than the weight and rope themſelves; for if they weighed more, the weight would ceaſe to deſcend, though it did not touch the bottom of the ſea.

Thus one may be deceived in meaſuring the length of a rope let down into the water, in order to determine the depth of the ſea; and therefore to prevent miſtakes, you had beſt tie to the end of the ſame rope, another weight heavier than the former, and if this weight does not ſink the rope deeper than the other did, you may reſt aſſured that the length of the rope is the true depth of the ſea: if it does ſink the rope deeper, you muſt tie a third weight yet heavier, and ſo on till you find two weights of unequal gravitation that run juſt the ſame length of the rope, upon which you may conclude that the length of the wet rope is certainly the ſame with the depth of the ſea.

SPORT LXXVII.

To repreſent lightning in a room.

THE room in which you are to repreſent lightning muſt not be large, but quite dark, and ſo very cloſe that the air cannot readily enter it. The room being thus in order, take a baſon into it with ſpirit of wine and camphyre, which muſt boil there till it is all conſumed, and nothing left in the baſon. This will rarify the camphyr, and turn it into a very ſubtile vapour, which will diſperſe itſelf all over the room; inſomuch that if any one enters the room with a lighted flambeau, all the impriſoned vapour will in a moment take fire, and appear as lightning, but without hurting either the room or the ſpectators.

Remark. Camphyr is of a nature ſo proper to retain and keep an unextinguiſhable fire, that it will burn entirely, and that very eaſily upon ice or among ſnow, which it melts, notwithſtanding their coldneſs; and if it be reduced to powder and thrown upon the ſurface of any ſtill water, and then lighted, it will produce a very pleaſant ſort of fire, for the water will appear all fire and flame; the reaſon of which I take to be, becauſe the camphyr is of a fat nature which reſiſts water, and of a light and fiery ſubſtance, which the fire graſps ſo keenly, that it is impoſſible for this ſubſtance to diſengage itſelf when once it is entangled.

SPORT LXXVIII.

To melt at the flame of a lamp a ball of lead in paper, without burning the paper.

TAKE a very round and ſmooth leaden ball, wrap it up in white paper, that is not rumpled, but clings equally about the ball without wrinkles, at leaſt as far as is poſſible; hold the ball thus wrapped up over the flame of a lamp or flambeau, and it will grow hot by degrees, and in a little time melt and fall down in drops through a hole in the paper without burning it.

SPORT LXXIX.

To repreſent an Iris or rainbow in a room.

EVERY one knows that the rainbow is a great arch of a circle that appears all on a ſudden in the clouds before or after the rain, towards that part of the air that is oppoſite to the ſun, by virtue of the reſolution of the cloud into rain; this arch is adorned with ſeveral different colours, of which the principal are five in number, namely red which is outermoſt, yellow, green, blue, and violet and purple, which is interior.

This Iris ſeldom appears alone, and is called the firſt and the principal rainbow, to diſtinguiſh it from another that commonly appears along with it, and for that reaſon is called the ſecond rainbow, the colours of which are not ſo lively as thoſe of the firſt, though they are diſpoſed after the ſame manner, but in a contrary order, upon which account a great many take it for a reflection of the firſt.

If you want to repreſent, at one time two ſuch Iris's in your room, put water into your mouth and ſtep to the window (upon which the ſun is ſuppoſed to ſhine) then turn your back to the ſun, and your face to the dark part of the room; and blow the water which is in your mouth, making it ſpurt out with violence into little drops or atoms; and among theſe little atoms or vapours, you will ſee by the rays of the ſun, two rainbows reſembling the two that appear in the heavens in rainy weather.

Oftentimes we ſee rainbows in water-works or ſpouts, when we ſtand between the ſun and the fountain, eſpecially when the wind blows hard, for then it diſperſes and divides the water into little drops. Which is full evidence, that the rainbow, which the philoſophers admire as much as the ignorant people do thunder, is formed by the reflection and refraction of the rays of the ſun darted againſt ſeveral little drops of water, that fall from the clouds in time of rain.

A rainbow may likewiſe be very eaſily repreſented, in a room with a window that the ſun ſhines upon, by a triangular priſm expoſed to the rays of the ſun, which in paſſing through the glaſs, will by their different reflections and refractions produce upon the wall or ceiling of the room, a very agreeable Iris, or at leaſt a texture of ſeveral different colours reſembling thoſe of the rainbow; and the further the ceiling or wall is diſtant, and the more it is dark, the colours will appear the more charming and lively. You may likewiſe imitate the colours of the rainbow by expoſing to the ſun a ſphere of chryſtal or glaſs, or a glaſs full of clean water.

SPORT LXXX.

To make a conſort of muſick of ſeveral parts with only one voice.

THE ſound conveyed diſtinctly to the ear, by remote bodies, againſt which the air is driven by the voice of an animal or otherwiſe, and then reflected, is what we call an echo; which is ſometimes double, triple, *&c.* when the voice is ſtrong enough to make ſeveral bodies, at different diſtances, beat back at ſeveral times the parts of the air to our ears, ſo that one echo is no ſooner ended than another begins.

Though moſt eccho's make us hear only the laſt words of the voice, becauſe the air, though ſtrongly impreſſed, has not the ſame force at the end that it had at the beginning; yet it may be ſo contrived as to make a conſort of muſic of ſeveral parts, that is, a conſort of ſeveral ſongs tuned together, by only one voice or one inſtrument, to the ſound of which the eccho anſwers.

For if the eccho anſwers only once to the voice or the ſound of the inſtrument, he who ſings or plays may make a Duo, that is, a muſic of two parts; and again a Trio or muſic of three parts, if the eccho anſwers twice. But indeed he muſt be an expert muſician, and one that is well verſed in varying the tune and the note.

Thus commencing, for example at Ut, he may begin Sol a little before the eccho anſwers, ſo as to finiſh the pronunciation of Sol by the time that the eccho has compleated its anſwer, and then he will have a fifth, which is a perfect conſonance in muſic; and in like manner, if at the ſame time with the eccho's anſwering to the ſecond note Sol, or a little before, he repeats it upon a higher or lower note, he will make a diapaſon or eighth, which is perfect harmony in muſic. And ſo on, if he has a mind to continue the chace with the eccho, and ſing alone the two parts.

To this purpoſe we ſee by experience in ſeveral churches, when they are ſinging, that there ſeems to be many more parts in the chorus than there really are, the quantity of eccho's making the air to reſound on all ſides, and ſo multiplying the voice and redoubling the chorus.

SPORT LXXXI.

To make a ſtring of a vial ſhake without touching it.

CHOOSE at pleaſure three ſtrings in a viol, or any other inſtrument of that ſort without any intermediating ſtring, and tune the firſt and the third to the ſame note, without touching that in the middle; then ſtrike one of the two ſtrings thus tuned pretty hard with a bow, and you will find that when it ſhakes the other will tremble ſenſibly and viſibly, and the middle ſtring, though nearer, ſhall ſtir no manner of way.

This Sport may likewiſe be reſolved by two ſtringed inſtruments of the ſame ſort, as two viols, two lutes, two harps, two ſpinets, *&c.* by putting the two in the ſame tune, and then placing them at a convenient diſtance, and in a proper poſition; for one of the two inſtruments being touched with a midling force, will move the other, that is, the ſtrings of the other, which are ſuppoſed to be in uniſon, will produce ſuch another harmony, eſpecially if the ſtrings in one and the other inſtrument are equally long and equally thick. For this I can aſſign no other reaſon but experience.

SPORT LXXXII.

To make a deaf man hear the ſound of a muſical inſtrument.

IT muſt be a ſtringed inſtrument, with a neck of ſome length, as a lute, a guitarre, or the like; and before you begin to play, you muſt by ſigns direct the deaf man to take hold with his teeth of the end of the neck of the inſtrument; for then if one ſtrikes the ſtrings with a bow one after another, the ſound will enter the deaf man's mouth, and be conveyed to the organ of hearing through the hole in the palate: and thus the deaf man will hear with a great deal of pleaſure the ſound of the inſtrument, as has been ſeveral times experienced. Nay, thoſe who are not deaf may make the experiment upon themſelves, by ſtopping their ears ſo as not to hear the inſtrument, and then holding the end of the inſtrument in their teeth while another touches the ſtrings.

SPORT LXXXIII.

To make an egg enter a vial without breaking.

LET the neck of the vial be never ſo ſtrait, an egg will go into it without breaking, if it be firſt ſteeped in very ſtrong vinegar, for in proceſs of time the vinegar does ſo ſoften it, that the ſhell will bend and extend lengthways without breaking. And when it is in, cold water thrown upon it will recover its primitive hardneſs, and, as Cardan ſays, its primitive figure.

SPORT LXXXIV.

To make an egg mount up of itſelf.

MAKE a little hole in the ſhell of the egg, and ſo take out the yolk and the white, and fill the egg-ſhell with dew, then ſtop up the hole and expoſe it to the rays of the ſun at noon-day; for then the dew not being able to bear the light, nor too great heat, will riſe up with the egg ſhell, eſpecially if it leans againſt a little ſtick or piece of wood, that ſlopes never ſo little, and if the hole is well ſtopped. May dew is ſaid to be beſt; and it is obſerved by the farmers, that the more May abounds in dew, the more plentifully does the earth bring forth: for dew being a ſubtile vapour produced in the morning by a weak heat, and preſerved by a moderate cold, it is very well diſpoſed for the reception of celeſtial virtues; and when it inſinuates itſelf into vegetables, it communicates to them the virtue it retains; and hence it comes that plants moiſtened with it, thrive better than when they are nouriſhed with ſpring, well, or river water.

SPORT LXXXV.

To make water freeze any time in a hot room.

FILL a vial with warm water, the neck of which is ſomewhat narrow, and having ſtopped it cloſe, put it in a veſſel full of ſnow, mixed with common ſalt and ſaltpetre, ſo as to leave the vial covered all over with ſnow: and in a little time the water will be quite frozen, though in the ſummer time, and in a very hot room.

If you throw cold water with ſnow upon a table, and upon the ſnow ſet a platter full of ſnow with a ſufficient quantity of ſalt and ſaltpetre pounded; the ſalt and the ſaltpetre will make the ſnow ſo cold, that in a little time the water under the platter will be turned to ice, and make the platter ſtick ſo faſt to the table, that you cannot move it without ſome difficulty.

Remark. The ſaltpetre and ſal-armoniac are likewiſe poſſeſſed of the virtue of making water ſo extremely cold, that if you put a ſufficient quantity of them in common water, it will become ſo cold that your teeth can ſcarce bear it. They might therefore be very uſefully employed in ſummer for cooling wine or any other liquor, by ſetting the wine bottles in water thus refrigerated.

If you diſſolve a pound of nitre in a pail of water, the water will be exceſſive cold, and ſo very proper for the uſes above-mentioned. It is well known that wine is cooled with ice; and in regard ice cannot always be had in ſummer, I ſhall here preſcribe a way of making it.

To make ice in ſummer.

To make ice in ſummer, put two ounces of refined ſaltpetre, and half an ounce of florentine Oris, into an earthen bottle filled with boiling water, ſtop the bottle cloſe, and convey it forthwith into a very deep well, and there let it ſteep in the well-water for two or three hours, at the end of which you will find the water in the bottle all ice: ſo you have nothing to do but to break your bottle and take out your ice.

SPORT LXXXVI.

To kindle a fire by the ſun-beams.

THIS Sport may be performed either by refraction in uſing lenticular glaſſes thicker in the middle than in the ſides, called burning-glaſſes, through which when the rays paſs they refract and unite into one point called the Focus, at which you may light a match or any other combuſtible matter: or elſe by reflexion, in uſing a concave looking-glaſs of metal well poliſhed in its concavity, which may be either ſpherical or parabolic, and is likewiſe called a burning-glaſs, but much better than the former ſort; for by it you may in a moment ſet fire to a piece of wood, and in a ſhort time melt lead, and even iron, and vitrify ſtone.

SPORT LXXXVII.

To make a fowl roaſting at the fire, turn round of itſelf with the ſpit.

TAKE a wren and ſpit it upon a hazel ſtick, and lay it down before the fire, the two ends of the hazle ſtick being ſupported by ſomething that is firm; and you will ſee with admiration the ſpit and the bird turn by little and little without diſcontinuing, till it is quite roaſted. This experiment was firſt found out by Cardinal Palotti at *Rome*, who ſhewed it to father Kircher, in order to know the phyſical cauſe of it; which to my mind is eaſily diſcovered, for the hazle wood is compoſed of ſeveral long and porous fibres, into which the heat inſinuates itſelf, and ſo makes it turn round when the wood is hung right.

SPORT LXXXVIII.

To make an egg ſtand on its ſmalleſt end, without falling, upon a ſmooth plain ſuch as glaſs.

PLACE a looking-glaſs quite level, or horizontally, without inclining to either ſide, toſs the egg with your hand till the yolk burſts, and the matter of it is equally diſperſed through all the parts of the white, ſo that the white and the yolk make but one body. Then ſet the end of the egg upon the horizontal plain, holding it till it is upright, and then it will continue in that ſituation without falling, by reaſon of the æquilibrium made on all ſides by the parts of the yolk equally mixed with the white, ſo that the center of gravity in the egg continues in the line of direction.

SPORT LXXXIX.

To make a piece of gold or ſilver diſappear, without altering the poſition of the eye or the piece, or the intervention of any thing.

PUT the piece of gold in a porringer full of water, or a veſſel that is broader than it is deep, and let the eye be in ſuch a poſition, as juſt barely to ſee the piece at the bottom over the brim of the veſſel; then take out the water, and though the porringer continues in the ſame poſition as well as the eye, the piece which appeared before by virtue of the refraction made in the water, will then be covered from the ſight by the ſides of the porringer.

SPORT XC.

To make a Loaf dance while it is baking in the Oven.

PUT into the dough a nut-ſhell filled with live ſulphur, ſaltpetre and quickſilver, and ſtopped cloſe; as ſoon as the heat comes to it, the bread will dance in the oven; which is occaſioned by the nature of quickſilver, for it can bear no heat without being in a continual motion. Thus, by the means of quickſilver put into a pot where peaſe are to be boiled, all the peaſe will leap out of the pot as ſoon as the water begins to heat. In like manner quickſilver put into hot bread, will make it dance up and down the table.

SPORT XCI.

To ſee in a dark room what paſſes abroad.

MAKE your room ſo cloſe and dark, that the light can come in no where but through a little hole left in a window upon which the ſun ſhines; over againſt this hole, at a reaſonable diſtane from it, place ſome white paper, or a piece of linen; and you will ſee every thing that paſſes by the outſide of the window appear on the paper or linen, only their figures are inverted.

SPORT XCII.

To hold a glaſs full of water, with the mouth down, ſo as that the water ſhall not run out.

TAKE a glaſs full of water, cover it with a cup that is a little hollow, inverting the cup upon the glaſs; hold the cup firm in this poſition with one hand, and the glaſs with the other, then with a jerk turn the glaſs and the cup upſide down, and ſo the cup will ſtand upright, and the glaſs will be inverted, reſting its mouth upon the interior bottom of the cup. This done, you will find that part of the water contained in the glaſs will run out by the void ſpace between the bottom of the cup and the brim of the glaſs; and when that ſpace is filled, ſo that the water in it reaches the brim of the glaſs, all paſſage being then denied to the air, ſo that it cannot enter the glaſs, nor ſucceed in the room of the water, the water remaining in the glaſs will not fall lower, but continue ſuſpended in the glaſs.

If you would have a little more water deſcend into the cup, you muſt with a pipe or otherwiſe draw the water out of the cup, to give paſſage to the air in the glaſs; upon which part of the water will fall into the glaſs till it has ſtopped up the paſſage of the air afreſh, in which caſe no more will come down; or, without ſeeking out the water in the cup, you may incline the cup and glaſs ſo that the water in the cup ſhall quit one ſide of the brim of the glaſs, and ſo give paſſage to the air, which will then ſuffer the water in the glaſs to deſcend till the paſſage is ſtopped again.

This Sport may likewiſe be performed by covering the brim of the glaſs that is full of water, with a leaf of ſtrong paper, and then turn the glaſs, as above; and without holding your hand any longer upon the paper, you will find it as it were glewed for ſome time to the brim of the glaſs, and during that time the water will be kept in the glaſs.

SPORT XCIII.

To make new Wine keep its ſweetneſs for ſeveral Years.

MR. LENTIN informs us, that if you let new wine heat of itſelf, it loſes in a little time all its ſweetneſs, eſpecially if the caſks are left open; but if you boil it upon a fire immediately after the grapes are preſſed, moſt of the volatile principles of the ſweetneſs concentrate, and link themſelves with the more fixed parts of the wine, which preſerves its ſweetneſs for ſeveral years.

Remark. A ſweet and new wine may preſerve its ſweetneſs at leaſt a whole year, if you pitch the caſk well both within and on the outſide, to hinder the water to penetrate into it, and ſo ſpoil the wine, which ought to be put into it before it boils; and keep the caſk well ſtopped in a ciſtern of water, ſo as to be covered all over for a month or thirty days; and then take out the caſk and place it in a cellar.

In the year 1692, I had a caſk full of Burgundy wine brought me in the ſummer to *Paris* by water, which immediately upon its arrival was clapped into my cellar, and after a few days ſtanding, I found it boiling as if it had been quite new, and that it had reaſſumed its former ſweetneſs, which continued about a month, and after that it proved extraordinary good wine. Some tell you that a piece of cheeſe or pumice-ſtone thrown into the caſk, will break the violence of fermenting wine.

To recover the ſweetneſs of new wine.

When the new wine has loſt its ſweetneſs, it may be recovered by caſking it up immediately, and putting in the bottom of the caſk half a pound of muſtard-ſeed, leſs or more, according to the ſize of the caſk.

SPORT XCIV.

To know when there is water in wine, and to ſeparate it from the wine.

IF the wine is neither ſweet nor new, but fine and clear of its lee, you may know, (according to Porta and father Schott) whether it is mixed with water or not, by throwing into it apples or pears, for if the wine is unmixed they will ſink to the bottom, if it is mixed they will ſwim above, becauſe the ſpecific gravity of water is greater than that of wine.

Some order wild apples or pears, and if theſe cannot be had, ripe apples or pears. Others make uſe of an egg, and alledge, that when the wine is pure the eggs falls ſwifty to the bottom, but if it is mixed with water, the egg deſcends more ſlowly, the water having, by virtue of its gravity more force to bear up the egg than the wine has.

Now the contrary will happen, if the wine be ſweet and new, that is, when ſuch wine is unmixed, the egg will deſcend ſlower than when it is mixed; by reaſon that new wine unmixed is by virtue of its lee heavier than water, and conſequently becomes lighter, by the addition of water.

When you have diſcovered that the wine is mixed, you may ſeparate the water by a dry bulruſh, according to Mizauld; for the ruſh being a plant that grows and thrives in watery marſhy places, if it be dried, and one end of it put into mixed wine, the water will inſinuate itſelf into the ruſh, and ſo the wine will be left alone. By the ſame reaſon, the ruſh may ſerve to diſcover whether the wine is mixed with water or not.

Remark. On the other hand, ſome pretend you may ſeparate the wine from the water, by putting in a long narrow piece of linen, woollen or cotton cloth, one end of which hangs out of the veſſel, as if the wine being lighter would riſe and flow out upon the cloth while the water ſtays behind; but this and ſeveral other ways for the ſame purpoſe, are diſapproved by other authors.

To pour water into wine without mixing.

You may pour wine upon water, without mixing, if you put a toaſt of bread upon the water in a glaſs, and while this toaſt ſwims above the water, pour in the wine very ſoftly; for then you will ſee the water remain unmixed at the bottom of the glaſs without any alteration in its colour.

To know if milk is mixed with water.

Here by the bye I ſhall ſhew you the way of knowing when water is mixed with milk; put a little ſtick into the milk, then pull it out, and let a drop of the milk fall from it upon the nail of your thumb; and if the milk is pure, the drop being thick will ſtand for ſome time upon your nail; but if it is diluted with water it will run off immediately.

To turn water ſeemingly into wine.

You may turn water into wine in appearance, by ſetting a vial full of water in a caſk full of wine, turning the mouth of the vial downwards; for then the water will run out, and the vial will be filled with wine; which the ignorant will take to be a turning water into wine.

SPORT XCV.

To make a metallic body ſwim above water.

THOUGH the ſpecific gravity of water is inferior to that of metals, and conſequently water is uncapable, abſolutely ſpeaking, to bear up a metallic body, ſuch as a ball of lead, yet this ball may be flatted and beat out to a very thin plate, which when very dry and put ſoftly upon ſtill water, will ſwim upon it without ſinking, by virtue of its dryneſs Thus we ſee a ſteel needle will ſwim upon the water, when it is dry, and laid ſoftly lengthways upon the ſurface of ſtill water.

But if you would have a metallic body to ſwim neceſſarily upon water, you muſt reduce it to a very thin plate, and that concave like a kettle, in which caſe the air it contains weighs leſs than the water, whoſe room it poſſeſſes. It is by this contrivance that copper boats or pontons are made for paſſing whole armies over rivers without any danger.

Remark. If you put this concave metallic veſſel upon the water, with its mouth perpendicularly down, it will ſtill ſwim, by reaſon that the air contained in its cavity finds no exit; inſomuch that if you puſh it under water and hold it there by force, the detained air will keep the bottom from being wet on the inſide. And by the ſame reaſon you may have a burning coal in the bottom, and find it not extinguiſhed when you take it out of the water, provided you do not hold it long under water, for fire ſtands in need of air to keep it in.

SPORT XCVI.

To make Aquafortis put up cloſe in a bottle boil without fire.

PUT a ſmall quantity of Aquafortis, and of the filings of braſs in a bottle, and you will ſee ſo great an ebullition, that the bottle will appear quite full, and be ſo hot that you cannot touch it without burning yourſelf.

Remark. In like manner if you mix oil of tartar and oil of vitriol together, you will preſently ſee a very great ebullition with a ſenſible heat, though neither of theſe liquors is compoſed of any combuſtible matter.

Of Aquafortis and Aqua Regia.

Aquafortis is ſo called with reſpect to its ſtrength, in diſſolving almoſt all metals and minerals. It is commonly a diſtillation from ſaltpetre and vitriol or green copperas; and it is yet better, if it be diſtillation from ſaltpetre and roach-allum. It diſſolves all metals but gold; but is rendered capable of the diſſolution of gold by diſſolving ſal-armoniac or ſea-ſalt in it, after which it aſſumes the name of Aqua Regia.

Of Sal-Armoniac, Roach Allum, and Saltpetre.

To avoid all obſcurity of terms; I ſhall here acquaint you by the bye, that Sal-armoniac is a compoſition of bay-ſalt, chimney-ſoot, and the urine of animals: That roach-allum is a mineral earthy ſharp ſalt, filled with an acid ſpirit, which is oftentimes found condenſated in the veins of the earth, or is taken from aluminous ſprings by evaporation; or is found among mineral ſtones, and diſengaged from them by diſſolution in water and evaporation. And in fine, that ſaltpetre is a ſalt that is partly ſulphureous and volatile, and partly terreſtrial; it is found in the dark cavernous places of the earth, and likewiſe in ſtables, by reaſon of the great quantity of volatile-ſalt in the urine and excrements of animals, which joins in with the ſalt of the earth by the continual action of the air.

Vitriol.

The oil of Vitriol (mentioned above) is a cauſtic oil diſtilled by a ſtrong reverberating fire from Vitriol. Now, Vitriol is a mineral ſalt, approaching to the nature of roach-allum, which is found chryſtaliſed in the earth of ſuch mines as abound in metals, which gives us to know that it contains in it ſome metallic ſubſtance, and particularly iron or copper. When it is loaded with copper, if you rub it againſt iron, it will ſtain it with a copper colour. But it is beſt for all manner of preparations when it partakes moſt of iron.

Tartar.

The oil of Tartar (mentioned above), is diſtilled from Tartar along with the ſpirit, from which it is ſeparated by a funnel lined with brown paper. Tartar itſelf is an earthy incorruptible ſubſtance, formed like a reddiſh cruſt round the inſide of wine-caſks, which thickens and congeals to the hardneſs of a ſtone, and is ſeparated from the pure parts of the wine, by the action of the fermentative ſpirit.

SPORT XCVII.

To make fulminating or thundering powder.

TAKE three parts of ſaltpetre, two parts of ſalt of tartar, and one part of ſulphur pounded and mixed together; heat in a ſpoon ſixty grains of this compoſition, and it will fly away with a fearful noiſe like thunder, and as loud as a cannon, breaking through the ſpoon and every thing underneath it, for it exerts itſelf downwards, contrary to the nature of gun-powder, which exerts itſelf upwards.

Salt of Tartar.

The ſalt of tartar, here uſed, is only a ſolution in water of the black ſubſtance that remains after the diſtillation of the oil of tartar, and an evaporation of that ſolution to a dry ſalt, which muſt be kept very cloſe, leaſt the moiſture of the air ſhould melt it.

SPORT XCVIII.

To make the Aurum Fulminans, or thundering gold.

PUT into a matraſs, upon hot ſand, the filings of fine gold, with a tripple quantity of Aqua Regia, which will diſſolve the gold: mix this ſolution with a ſextuple quantity of ſpring water, and then pour upon it, drop by drop, the oil of tartar or volatile ſpirit of ſal-armoniac, till the ebuillition ceaſes, and the corroſion of the Aqua-regia is over, for then the powder will precipitate to the bottom, which may be dulcified with warm water, and dried with a very ſlow fire.

This powder is much ſtronger than the laſt deſcribed; for if you ſet fire to 20 grains of it, it will act with more violence, and have a louder crack than half a pound of gun-powder, and two grains of it kindled at a candle have a ſtronger report than a muſket ſhot.

SPORT XCIX.

To make the Sympathetic-Powder.

THE Sympathetic-powder is nothing elſe but the Roman vitriol calcined and reduced to a white light powder, which is ſaid to cure wounds at a diſtance, by being put upon a linen cloth dipped in the wounded perſon's blood, or upon a ſword whereon is the blood or pus that comes out of the wound. This cloth or ſword is wrapped up in a white linen cloth, which is opened every day, in order to ſtrew ſome freſh powder upon the blood or pus of the wound. This courſe they continue till the wound is perfectly cured, which happens the ſooner, if the cloth upon which is the blood and the powder, is kept in a place that is neither too hot nor too cold, nor too moiſt. Nay, it is neceſſary ſometimes to ſhift the cloth from place to place, according to the different diſpoſitions of the wound, by putting it, for example, in a cold place, when the patient finds an exceſſive heat in the wound.

To calcine the vitriol for the ſympathetic powder, take ſome Roman vitriol, when the ſun is in the ſign of Leo, or in the month of July, diſſolve it in rain water, and filtrate the water through ſinking pater. Then let the water evaporate upon a gentle fire, and you will find at the bottom the vitriol in little hard ſtones of a fire-green colour. Spread theſe ſtones carefully, and expoſe them to the rays of the ſun, ſtirring them often (with a wooden ſpatula; not an iron ſpatula, becauſe the ſpirits of the vitriol are ready to join in with iron, which would rob the ſympathetic powder of its volatile ſpirits, in which all its virtue conſiſts) that the ſtones may be the better penetrated by the ſun, and calcined and reduced to a powder, which will be as white as ſnow. And to render the ſubſtance of the vitriol more pure and homogenous, the diſſolution, filtration, coagulation and calcination ought to be repeated three times.

This wonderful powder muſt be carefully kept in a vial cloſe ſtopped, and in a dry place, for the leaſt moiſture of the air may turn it to vitriol again, and ſo make it loſe its ſympathetic virtue.

We are told that this powder ſtops all bleedings, and mitigates very much all ſorts of pains in any part of the body, particularly the toothach; and that, by application not to the part affected, but to the blood taken from it, and covered up in a linen cloth as above.

Remark. The chymiſts have another calcination of vitriol called Colcothar, which being put into the noſe ſtops a bleeding at the noſe, and provokes to ſneeze; being of ſovereign uſe in rouzing the ſenſes, wherefore it is given in lethargies. It is alſo ſucceſsfully uſed for drying up wounds and ulcers. This colcothar is only the vitriol kept melted upon a fire till all its humidity is evaporated, and it is reduced to a hard reddiſh brown maſs, whereby it is rendered fit for the cure of the aforeſaid maladies, and many others not here mentioned.

SPORT C.

Of the magnetical cure of diſeaſes by tranſplantation.

THE magnetic cure by tranſplantation, is, that which is performed by communicating the diſeaſe to ſome beaſt, tree, or herb; and, as ſome will have it, is founded upon the efflux of the morbific particles, which paſs by inſenſible tranſpiration out of the body of the patient into another animal or plant.

Froman informs us, that a young ſtudent got rid of a malignant fever by giving it to a dog that lay in the bed with him, and died of it: which if true, muſt needs proceeds from the inſenſible tranſpiration of the ſubtile matter, that thereupon entered the pores of the dog.

Thomas Bartholin ſays, his uncle was cured of a violent cholic, by applying a dog to his belly, which was thereupon ſeized with it; and that his maid-ſervant was cured of the toothach by clapping the ſame dog to her cheek, and when the dog was gone from her he howled and made ſuch motions, as gave them to know he had the maid's tooth ach.

Hoffman ſpeaks of a man cured of the gout by a dog lying in the bed with him, who thereupon was ſeized with it. And frequently after the dog had fits of the gout, as his maſter had uſed to have before. However this be, certain it is, that dogs are often ſubject to the gout, without any infection from men; and this, and the other ſtories of tranſplantation are not here offered for concluſive proofs, but by way of recreation.

Monſieur de Valemont, who ſeems inclinable to believe tranſplantation of diſeaſes, ſays, it is done not only by inſenſible tranſpiration, but likewiſe by ſweat, by urine, by the blood, by the hair, or by taking up what falls from the ſkin, upon a ſtrong friction. For this he brings ſeveral inſtances, and particularly that which follows:

A perſon of quality in *England*, uſed to cure the jaundice at a great diſtance from the patient, by mixing the aſhes of aſh-wood with the patient's urine; and making of that compoſition three or ſeven, or nine little balls, with a hole in each of them, in which he put a leaf of ſaffron, and then filled it up with the ſame urine. This done, he hung theſe balls in a private place where no body could touch them; and from that time the diſeaſe began to abate.

The great virtues of the Aſh-tree.

Remark. The Aſh, which is a common tree all over Europe, has merited the appellation of the Vulnerary wood, by reaſon of its peculiar property in curing ſeveral diſeaſes, and above all wounds and ulcers. Not to mention the almoſt incredible virtues aſcribed to it, it is ſaid to ſtop bleeding at the noſe, if the face be but rubbed with the wood, and then rubbed with fair water, and if the patient holds in the hand of that ſide where the bleeding is, a piece of the wood till it heats his hand.

SPORT CI.

To ſtop a bleeding at the noſe, or at any other part of the body.

FATHER Schott the Jeſuit ſays, that to ſtop a bleeding at the noſe, you need only to hold to the noſe the dung of an aſs very hot, wrapped up in an handkerchief, upon the plea that the ſmell will preſently ſtop it ——Wecher did the ſame with hogs dung, very hot done up in fine taffata, and put into the noſe.

I have ſeveral times experienced, that a piece of red coral held in the mouth, will ſtop a bleeding at the noſe. Some tell you that the conſtriction of the thumb of the ſide of the noſtril that bleeds, will do the buſineſs.

To ſtop the bleeding of a wound, take a linen cloth in the ſpring when the frogs lay their eggs in the water, and waſh it in that water till it is well impregnated with the frogs eggs; then dry it at the ſun; and after repeating this impregnation and deſiccation three or four times, keep the cloth to be applied to the wound twice in the form of a cataplaſm. We are told the ſecond application will do.

SPORT CII.

To prepare an ointment that will cure a wound at a diſtance.

THE ointment mentioned by Paracelſus is prepared thus, according to Goclenius: Take of the Uſnea or moſs of the ſcull of a man that was hanged, two ounces; mummy, human blood, of each half an ounce; earth-worms waſhed in water or wine, and dried, two ounces and a half; human fat, two ounces; the fat of a wild boar, and the fat of a bear, of each half an ounce; oil of linſeed and oil of turpentine, of each two drachms.

John Baptiſt Porta preſcribes it a little otherwiſe by throwing in ſome bole armoniac, and leaving out the earth-worms, and the bears and boars fat. But let the compoſition be which it will, it muſt be well mixed and beat in a mortar, and kept in a long narrow vial. Some ſay it ſhould be made when the ſun is in Libra.— The way of uſing it is this:

Put into the ointment the weapon or inſtrument that gave the wound, and leave it there; then let the patient waſh his wound every morning with his own urine, and apply nothing elſe to it; after it is well waſhed and cleanſed, let him tie it up tight with a clean white linen cloth, and he will find it will heal without any pain.

Monſieur Vallemont ſays, if you cannot get the inſtrument with which the wound was given, you may take another, which if gently conveyed into the wound, and impregnated with the blood and animal ſpirits reſiding there, will have the ſame effect. He adds, that if you want a ſpeedy cure, you muſt anoint the inſtrument often, otherwiſe you may let it lie a day or two without touching it.

The effect of this unguent he imputes to the ſubtle particles, which are theſe little agents that diſengage themſelves from the moſt ſpirituous and tranſpirable ingredients of which this unguent is compoſed.

To add to the credibility of its operation, he quotes father Lana, who obſerved that when the vines in *France* were in flower, the wines in *Germany*, though at a great diſtance, ſuffer an efferveſcence; which he explained by the effluviums of ſubtile matter, making theſe to reach as far as the ſtars, and alledging that if the attoms, which tranſpire from the terreſtrial globe, were not carried to the ſtars, and ſent back from the ſtars to the earth by a perpetual flux and reflux, there would be no phyſical commerce between the heavens and the earth.

SPORT CIII.

To pierce the head of a Pullet with a needle without killing it.

THIS is a very eaſy Sport, for there is a place in the middle of a pullet's head, that may be pierced without hurting the Cerebellum.

bellum. But the needle muſt not be kept in above a quarter of an hour.

SPORT CIV.

To make handſome faces appear pale and hideous in a dark room.

BURN ſome brandy and common ſalt in a glaſs, then put out the fire and all the lights in the room; and the particles of the ſalt and brandy evaporating into the air ſhut up in the room, will make the faces of the people in the room appear through that air hideous and frightful.

I intimated above, that if, inſtead of brandy, you take good ſpirits of wine mixed with camphyr in a glazed earthen pan put upon hot burning coals, he that enters the room with a lighted candle will be agreeably ſurpriſed; for the candle ſetting fire to the particles of the ſpirit and the camphyr, with which the air is repleniſhed, that air will ſeem to be all in a fire and the perſon will ſee himſelf in the midſt of flames without being burned.

SPORT CV.

To lift a mortar of ten pounds weight with a wine-glaſs.

TURN the mortar up-ſide down, rub the bottom very ſmooth, ſo that there be not the leaſt unevenneſs; make a circle the bigneſs of the rim of the glaſs with clay uſed by diſtillers; then take a lighted paper, put it into the glaſs, turn it down on the mortar, fixing it to the circle of clay and luting it quickly with the ſame, ſo as not the leaſt air may enter: when the paper is burned out, and the glaſs is cool, proceed to the experiment and you will find it anſwer.

SPORT CVI.

To make an egg move upon a table.

TAKE an egg, make a ſmall hole at each end, and after you have blown out the contents through theſe holes, put a live leech into it, then ſtop up the holes with wax: the egg thus prepared being ſet upon the table, place ſome freſh water at a little diſtance, which the leech immediately ſmelling will make towards, and give a motion to the egg, to the agreeable ſurprize of the ſpectators.

SPORT CVII.

To make an artificial flame.

ANY of the following oils, mixed with compound ſpirits of Nitre, burſt into a flame, *viz.* oil of Carroways, Cloves, Saſſafras, Guiacum, Box, Camphyr, Pepper, Hartſhorn, Blood, and many other oils.

SPORT CVIII.

To make an artificial earthquake.

TO twenty pounds of iron filings add as many pounds of ſulphur; mix, work and temper the whole together with a little water, ſo as to form a maſs, half moiſt and half dry: this being buried three or four foot under ground, in ſix or ſeven hours time will have a prodigious effect; the earth will begin to tremble, crack and ſmoke, and fire and flame burſt through.

SPORT CIX.

To prepare a lamp that will make a company look of any colour you pleaſe.

IF you would have a green aſſembly, order a lamp to be made of green tranſparent glaſs, put green oil and a green wick in it, and the company will appear green. If you would have a black company, you muſt do the ſame in black and ſo in any colour.

SPORT CX.

To bake an egg upon your head.

TAKE a loaf of bread juſt from the oven, cut a hole in it, put the egg in this hole and cover it with the bread cut out, then wrap the loaf in a linen cloth, and hold it upon your head for ſome time, the egg will be ſoon done enough.

SPORT CXI.

To make flames iſſue from an egg.

BLOW out the contents of an egg and dry the ſhell in the ſun, then fill it with brimſtone, ſaltpetre, and unquenched lime; cloſe the holes of the egg through which you blew the contents, then throw it into water and it will flame.

SPORT CXII.

To make a ring leap about.

PREPARE a ring made of copper, hollow, fill it with quickſilver, and ſtop cloſe the hole again that the quickſilver may not find any vent. This ring put upon an iron plate made very hot with coals underneath, will ſkip about briſkly.

SPORT CXIII.

Of a deceitful bowl to play withal.

MAKE a hole in one ſide of the bowl, and caſt melted lead therein, and then ſtop up the hole cloſe, that the knavery or deceit be not perceived; you will have pleaſure to ſee that notwithſtanding the bowl is caſt directly to the play, how it will turn away ſideways; for that on that part of the bowl which is heavier upon one ſide than the other, it never will go truly right, if artificially it be not corrected, which will hazard the game to thoſe who know it not; but if it be known that the leady ſide in rolling, be always under or above, it may go indifferently right; if otherwiſe, the weight will carry it always ſideways.

SPORT CXIV.

To contrive a veſſel which keeps its liquor when filled to a certain height, but loſes or ſpills it all when filled a little fuller with the ſame liquor.

TAKE a glaſs and run through the middle of it a ſmall bended pipe or crane, open at the end next the bottom of the glaſs, and likewiſe at the other end, which muſt be lower than the bottom of the glaſs; for then the water or wine poured into the glaſs continues in it while the branch is filling, and till it comes to the bend or the uppermoſt part of the crane, which withal ſhould be a little lower than the upper edge of the glaſs: but after that if you continue to pour more in, it will riſe higher in the concavity of the glaſs, and not finding place for a farther aſcent into the crane by reaſon of its bending downwards, it will change its aſcent into a deſcent through the branch, and continue to deſcend and run out by the end, as long as you continue to pour in; nay, when you have done pouring, you will ſee that all that was in the glaſs before is gone.

You may make the water run out at the lower end, though the glaſs is not filled up to the top of the crane, namely, by ſucking at the lower aperture the air contained in the crane, for then the water will neceſſarily ſucceed in the room of the air, and continue to deſcend through the branch, till the glaſs is empty, eſpecially if the orifice touches the bottom of the glaſs.

Or elſe, run the ſmall pipe perpendicular down through the glaſs; let the pipe be open at both ends, the uppermoſt of which ought to be a little lower than the brim of the glaſs, and the other end a little lower than the bottom of the glaſs. Put this ſmall pipe in another larger pipe ſtopped at the upper end, which muſt be a little higher than the end of the firſt and ſmaller pipe, and open at the lower end, which muſt touch the bottom of the glaſs if you would have all the water run out, which it will do when it riſes, for then paſſing through the orifice of the pipe, it will enter the pipe by one end and run out at the other.

SPORT CXV.

To make a coach that a man may travel in without horſes.

THE two four-wheels muſt be little, and moveable round their common Axeltree, as in the ordinary coachess and the hinder wheel muſt be large, and firmly fixed to their common axeltree, inſomuch that the axeltree cannot move, without the wheels move along with it.

Round the middle of the axeltree put a trundle-head, with ſtrong and cloſe ſpindles, and near to that fix upon the beam a notched wheel, the notches of which may catch the ſpindles of the trundle-head, and ſo in turning with the handle, that wheel round its axeltree, which ought to be perpendicular to the horizon, it will turn the trundle, and with that the axeltree and the wheels, which will thereupon ſet forward the coach, without horſes or any other animal. I need not tell you that the axeltree muſt enter into the beam, in order to turn within it.

There was invented at Paris ſome years ago, a coach or chaiſe, which a footman behind it made to go with his two feet alternately, by virtue of two little wheels hid in the box between the two hind wheels, and made faſt to the axeltree of the coach.

In ſhort, the contrivance of the machine is this: In a roller, the two ends of which are made faſt to the box behind the chaiſe, is a pully upon which runs the rope that faſtens the end of the planks, upon which the footman puts his feet, where is a piece of wood that keeps faſt the two planks at the other end, allowing them to move up and down by the two ropes, tied to their two ends, where there are two little plates of iron which ſerve to turn the wheels that are fixed to their axeltree, which is likewiſe fixed to the two great wheels.

Thus, you will readily apprehend that the footman putting his ſeet alternately upon the end of each plank, one of the plates will turn out of the notched wheels; for example, if he leans with his foot upon one plank, it deſcends and raiſes the other, which can't riſe but at the ſame time the plate of iron that enters the notches of the wheel, muſt needs make it turn with its axeltree, and conſequently the two great wheels. Then the footman, leaning upon one plank, the weight of his body will make it deſcend and raiſe the other plank, which turns the wheel again; and ſo the motion will be continued.

'Tis eaſy to imagine that while the two hind-wheels advance, the two fore-wheels muſt likewiſe advance; and that theſe will always advance ſtraight if the perſon that ſits in the chaiſe manages them with reins made faſt to the fore-beam.

SPORT CXVI.

To make two Images, one ſhall light a candle, and the other blow it out.

UPON the ſide of a wall make the figure of two images, in the mouth of each put a pipe or quill, ſo artificially that it be not perceived, in one of which place ſaltpetre very fine and dry and pulveriſed, and at the end ſet a little match of paper: in the other quill ſulphur beaten ſmall. Then, holding a lighted candle in your hand, ſay to one of thoſe images by way of command, *Blow out the candle*, then lighting the paper with the candle, the ſaltpetre will blow out the candle immediately, and going to the other image (before the ſnuff of the candle be out) touch the ſulphur with it, and ſay, *Light the candle*, and it will immediately be lighted.

SPORT CXVII.

To ſeem to turn water into wine.

TAKE four beer-bowl-glaſſes, rub one in the inſide with a piece of allum; let the ſecond have a drop of vinegar in it, the third empty, and the fourth as much clean water in it as your mouth will contain: have ready in your mouth a clean rag with ground braſil tied cloſe in it, that the bulk may be no bigger than a ſmall nut, which muſt lie betwixt your hind teeth and your cheek; then take off the water out of the glaſs into your mouth, and return it into the glaſs that hath the drop of vinegar in it, which will cauſe it to have the perfect colour of ſack; then turn it in your mouth again and chew your rag of braſil betwixt your teeth, and ſquirt the liquor into the empty glaſs, and it will have the perfect colour and ſmell of claret; returning the brazil into its former place, take the liquor into your mouth again, and preſently ſquirt it into the glaſs you rubbed with allum, and it will have the perfect colour of mulberry-wine; and ſo in many other ways, which for brevity I omit.

SPORT CXVIII.

To deceive one with ſeeming pieces of tobacco-pipe.

ROUL up a piece of white paper as hard as your lottery tickets, till it is as thick as a tobacco-pipe, then faſten the outward edge with a little ſtarch or paſte, having ſo done, cut the ends even; have this in your hand, break two pieces in the ſight of the company, ſhake the three together in your hat, then caſt them upon the table, ſaying, How many pieces of pipe is there under the hat? Every one will be apt to ſay, three; liſt up the hat the better to urge them, clapping it down preſently, ſaying, now I'll hold you a wager there are but two pieces of pipe under the hat; which when laid, take up the hat, and their folly will ſoon be diſcerned, by your cutting the paper with your knife.

SPORT CXIX.

To write letters ſecretly, that cannot be diſcover'd

TAKE a ſheet of white paper, and double it in the middle, then cut holes through both the half ſheets, let the holes be cut like the panes of glaſs windows, or other forms what you beſt fancy, and then with a pin prick two little holes at each end and cut your paper in two halves, give one half to your friend (to whom you intend to write) the other half keep to yourſelf: now when you do write, lay your cut paper on a half ſheet of writing paper, and ſtick two pins through the two holes that it ſtir not; then through thoſe holes that you did cut, write your mind to your friend; when you have done, take off your paper with the holes again, and then write ſome other idle words both before and after your lines; but if they were written to make ſome little ſenſe, it would carry the leſs ſuſpicion; then ſeal it up and ſend it.

When your friend hath received it, he muſt lay his paper on the ſame, putting pins into the pin holes, and then he can read nothing but your mind which you writ, for all the reſt of the lines are covered.

SPORT CXX. *Secret Writing.*

WRITE a letter (what you pleaſe) on one ſide of paper with common ink, then turn your paper, and write on the other ſide with milk, (that which you would have ſecret) and let it dry; but this muſt be written with a clean pen: Now when you would read it, hold that ſide which is written with ink to the fire, and the milky letters will then ſhew blewiſh on the other ſide, which may be perfectly diſcerned.

SPORT CXXI. *Secret Writing.*

PUT the powder of roach allum into a little water, and with that write upon the paper: when the letters are dry they will diſappear; but clap the paper in fair water, and the letters will look white and ſhining, the paper being a little blacked with the allum.

SPORT CXXII. *Secret Writing.*

WRITE with the water in which ſal-armoniac well pulveriſed is diſſolved: when the letters thus written are dry, they will diſappear, but hold them near the fire, and then they become viſible again. The ſame may be performed by writing with the juice of a lemon, or an onion.

SPORT CXXIII.

To make Glaſs-Drops.

GLASS-DROPS are thick little pieces of glaſs, made almoſt like a drop which have a long ſlender end, which being broken at its extremity, the drop breaks preſently with a crack, and flies into white powder and little fragments to two or three foot round.

Theſe drops, which have excited the curioſity, and perplexed the reaſon of moſt philoſophers, are made by letting a little of the melted matter of which the ordinary glaſſes are made, fall into a veſſel full of cold water; for then this melted matter which is very glutinous while 'tis red, makes a long ſtring, by which they hold the drop in the middle of the water, where it cools and hardens in a little time; after which they ſeparate the ſtring which is out of the water, ſo that the remaining part in the water does not break, commonly called a glaſs-drop, to this drop there ſticks a ſmall end, part of which may be ſeparated, by making it red at the flame of a candle, without breaking the drop; nor will this drop break if you lay it upon wood, and with a hammer ſtrike upon its thickeſt part, for its external parts are very hard, and ſupport one another like a vault. And they only break upon bending the ſlender end till it breaks, by virtue of the ſpring raiſed by that effort in all its parts, which ſhake and tremble like an extended ſtring, put into motion by forcing it to bend; whence it comes that theſe parts do in a little time return with very great veloſity to their firſt diſpoſition; and that the parts which are leſs united, and only contiguous, as it were, diſunite and ſeparate, and that occaſions the diſunion and ſeparation of all the reſt, and their flying all about with a noiſe.— See upon this head Mr. MARIOTTE's diſcourſe of the nature of the air publiſhed in 1679, in which he has, in my opinion, wrote more pertinently of this ſubject, than any one beſides.

SPORT CXXIV.

To repreſent the four Elements in a vial.

THE four elements of which the author of nature has compoſed the elementary world are the *earth*, *water*, *air* and *fire*; of which, the earth being the heavieſt, is ſaid to have the lowermoſt ſtation in the centre of the world: water being lighter covers the earth; air being lighter than water covers it; and at laſt fire the lighteſt of all ſurrounds the air. So that in this ſenſe theſe four make four concentrical orbs, the common center of which is the center of the world.

We may repreſent the four elements in this order: in a long vial of glaſs or chryſtal, by the help of four heterogeneous liquors, that is liquors of a different ſpecific gravity, which are of ſuch qualities, that, though ſhaked together by a violent agitation, they ſoon after return to their natural ſtations, and all the particles of one and the ſame liquor unite in a ſeparate body from the reſt, the lighter giving way to the heavier.

To repreſent the earth, make uſe of crude antimony, or blue ſmalt well refined, or black ſmalt coarſly pounded, which by its weight will ſink to the bottom of the vial.

To repreſent water, pour upon the laſt the terreſtrious ſubſtance of the ſpirit of tartar, or calcined tartar, or the clean ſolution of pot-aſhes, with a little roach-azur, which will give a ſea colour.

To repreſent the air, pour upon this compoſition ſpirit of wine rectified three times, till it has a colour of air, or elſe the moſt ſpirituous brandy with a little turnſol, which will give it a celeſtial blew or air colour.

To repreſent fire, pour upon all three the oil of Belin, which by its colour, likeneſs and ſubtilty, will make a pretty near reſemblance.

SPORT CXXV. *A Trick.*

TAKE a ball (or any thing elſe) in each hand, and ſtretching them as far as you can aſunder, lay a wager with any perſon, that without bringing your hands nearer together, or throwing either ball from one hand to the other, you will have them both in which hand the perſon pleaſes who lays the wager with you: which is no more than to lay one ball down upon the table, and turning yourſelf round take it up with the other hand, without bringing your hands any cloſer.

THE END.

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